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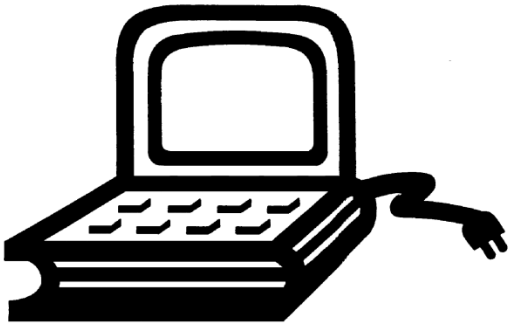


Math 150 – Fall 2019
Algebra & Trigonometry

Charles Rubenstein, Ph. D.
Professor of Engineering & Information Science

Preface, Session 1: Monday 8/26/19

6:30pm - 9:20pm

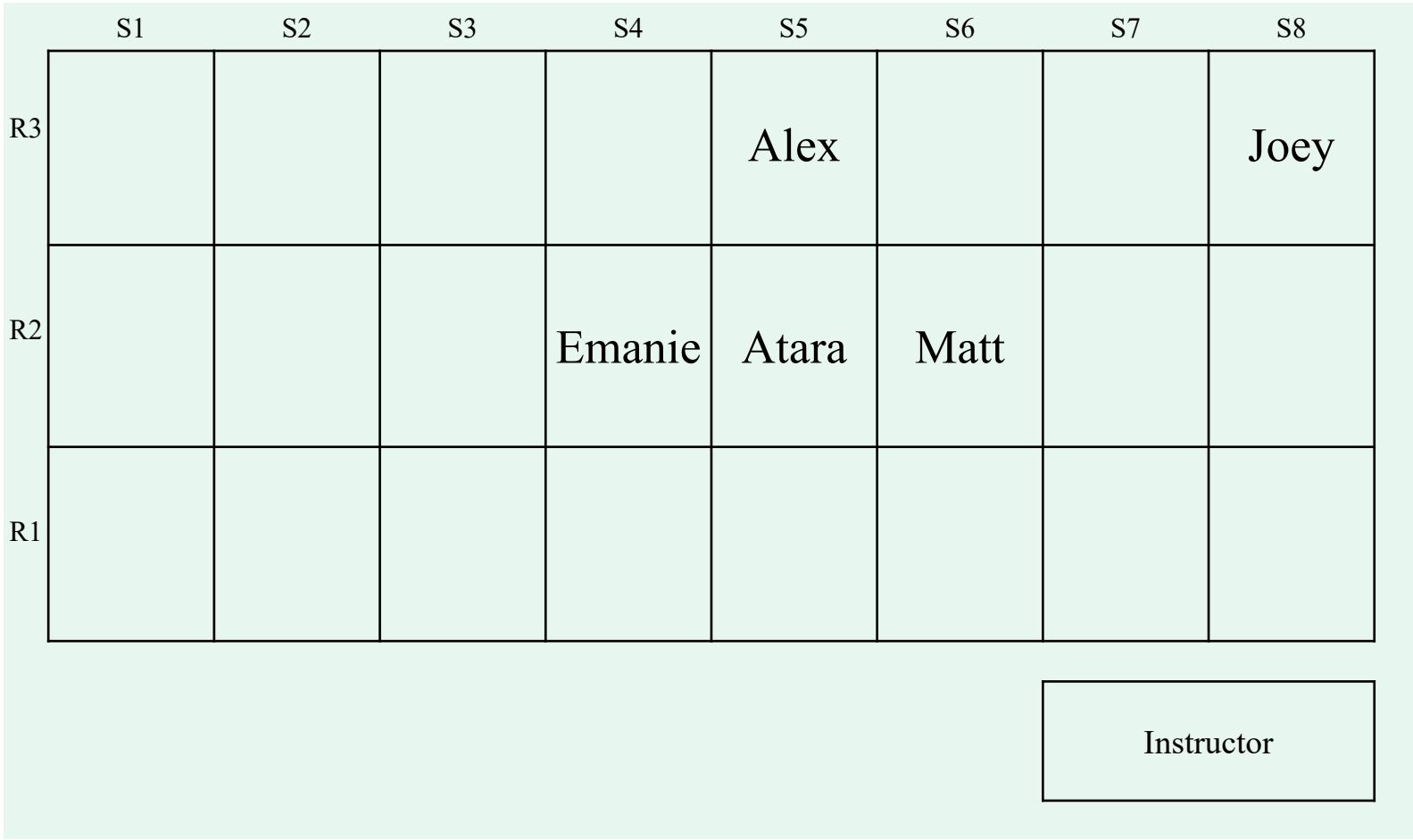
PMC402

Not Permitted in Class

Be sure to have all cellphones **OFF**

150-01 Class Seating Chart – Mondays

MONDAY
6:30pm



Instructor Contact Information

Dr. Charles Rubenstein <crubenst@pratt.edu>

Professor of Engineering & Information Science

Brooklyn Campius Faculty Office: ARC G-49

Pratt Manhattan Campus – PMC 402

Fall 2019 Office hours

- Mondays: **4:00-5:00pm ARC; 5:30-6:30pm PMC**
- Tuesdays: **5:00-6:00pm ARC**

*(*Please email me at least a day in advance if you plan on coming to office hours…)*

Send me an email … crubenst@pratt.edu

Subject line: 150 – or – Math

Who is *Dr. Rubenstein ?*

- Subject Background in
  - Bioengineering
  - Electrical Engineering
  - Systems Analysis
  - Information Science
- Certifications
  - Microsoft Trainer
  - CompTIA A+ Certified
- Professional Society Memberships
  - **ALISE** (*Member*), **IET** (*Fellow, Chartered Engineer*)
  - **IEEE** (*Senior Member) Member of the 2010-2011 Board of Directors of this 430,000 member professional engineering organization that produces more than one third of the world's electrotechnology information*

But… It's really all about …You!

Your Pratt Education … is founded on choices of

- **Career**
- **Topics of Study**
- **Background**

My Style of Teaching Math 150

Each class will consist of a mix of

- *Lectures*
- *In-class Review of Homework Assignments*
- *Small team problem solving*
- *Interactive Lectures* = **Class participation**

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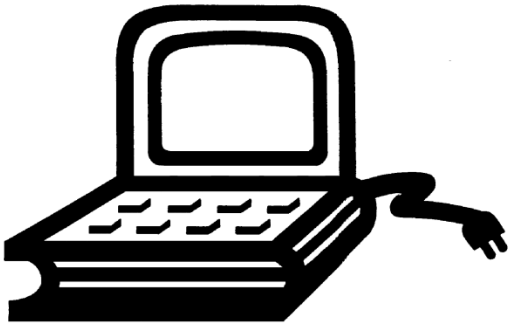

Math 150

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Syllabus Review

Algebra & Trigonometry

In Today's Class:

- *Download Textbook Chapter 1*
- *Review the Syllabus*
- *Review Course Schedule*
- *Textbook and Calculator needs*
- *Review of the Class Archive Concept (Homework online)*
- *Lecture and in-class problem solving*

For our next class – Session 2:

- *Begin readings, homework*
- *DUE: Homework Set #01*
- *Homework Review Set #01*

- *NOTE: Quiz #01 during class Session 3*

Math 150 – Class Topics

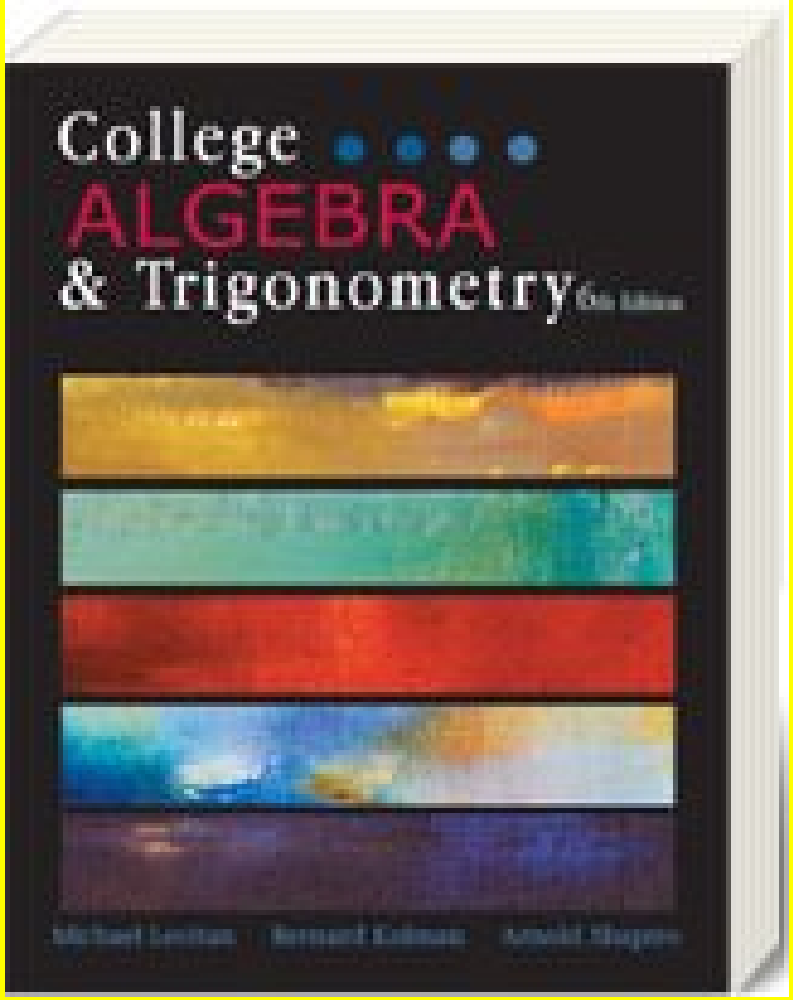
1. **The Foundations of Algebra**
2. **Equations and Inequalities**
3. **Functions**
4. **Polynomial Functions**
5. **Rational Functions and Conic Sections**
6. **Exponential and Logarithmic Functions**
7. **The Trigonometric Functions**
8. **Analytic Trigonometry**
9. **Applications of Trigonometry**
10. **Systems of Equations and Inequalities**
11. **Matrices, Linear Systems, and Determinants**
12. **Topics in Algebra**

Draft Schedule: Math 150 – Fall 2019 – PMC 402

Monday	Notes
26-Aug	1. Introduction: Numbers, Arithmetic Operations, Fractions
2-Sep	*Pratt Holiday - NO CLASSES – Labor Day*
9-Sep	2. Manipulation of Algebraic Expressions
16-Sep	3. Solving Linear and Quadratic Equations of One Variable (Q1)
23-Sep	4. Solving Equations of Two Variables (Q2)
30-Sep	*NO CLASSES – Instructor Holiday*
7-Oct	5. *1-hour in class Exam;* Creating Equations – Polynomials (Q3)
14-Oct	6. Polynomial Functions, continued (Q4); *1st Exam Review*
21-Oct	7. Functions, Graphing, Exponents and Logarithms (Q5)
28-Oct	8. Trigonometric Functions, Pythagorean Theorem (Q6)
4-Nov	CM/FM Seminar / 9. Applications of Trigonometry (Q7)
11-Nov	10. Analytic Trigonometry, Identities, Graphing (Q8) *Take Home Exam*
18-Nov	11. Areas and Volumes of Geometric Solids (Q9) *2nd Exam Due*
25-Nov	12. Systems of Equations and Inequalities (Q10) *2nd Exam Review*
2-Dec	13. Series and Sequences, Review topics
9-Dec	– *Conflict Day* – Final Examination (3-hour)

Questions?

Recommended Textbook:



Pub Date: 2011

Publisher: BVT Publishing

ISBN: 978-1-60229-880-4

($45.00)

College Algebra & Trigonometry

Sixth Edition

Michael Levitan, Bernard Kolman, and Arnold Shapiro

Recommended:

ANY Graphing Calculator (hp, TI, etc.)

Any graphing calculator may be used in this class

I have a TI-83 Plus

Mr. Ramus has loaner units if you don't want to buy one

Your Grade in Math 150

Homework Quizzes (25%)

Two Exams (40%)

Final Exam (30%)

Class Participation (5%)

About Homework Assignments

Homework is an essential part of this class.

Doing the assignments will allow you to keep up with the class and your studies, BUT, *Homework will NOT be collected.*

There are TWELVE (12) Homework assignments.

There will be a **5 minute in class quiz** on the first ten (10) Homework assignments (worth 2.5% each) *ONE WEEK AFTER THEY ARE REVIEWED* for a total of **25%** of your final grade.

There will be NO make up quizzes

About Your Exams

- Two (2) exams worth 20% and 15% each = **35%**

  (a one-hour in class exam and a two-hour take home exam - to make up for class schedule challenges)

- An in-class FINAL exam worth **35%**

These are Open Book exams

Formula will be provided, no need to memorize…

Examinations are designed to see what you have learned this semester as well as

to see what you DIDN'T learn

and prepare you for your next math class

There will be NO make up exams

All Materials are available on the Class Session Archive at:

www.CharlesRubenstein.com/150

www.CharlesRubenstein.com/150

- **syllabus.pdf = Syllabus**
  = ***includes revised Class Schedule***
- **Levitan6ed_ch1.pdf**
  = ***6th Edition Textbook's First Chapter***
- **HWK1to6.pdf = *Homework Sets #01-#06***
- **HWK7to12.pdf = *Homework Sets #07-#12***
- **19fa01.pdf** = *This slide set**
- **19fa01_h.pdf** = *slide set as handouts**

**Available by Thursday evenings…*

Questions?

Pratt

Math 150

Charles Rubenstein, Ph. D.
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Chapter 1 – Part 1

Foundations of Algebra

On Memorization of Formulas

You are NOT required to memorize anything.

When we do the proof of the Pythagorean Theorem you should want to memorize it unless you are already able to write down an equivalent proof.

All exams will be open book, open notes.

You are responsible for writing all formulas and definitions (sine, cosine, and tangent) on a ***SINGLE sheet*** in your notebook if you do not have them memorized.

After seeing these formulas enough times, you'll quite likely unconsciously memorize them.

Chapter 1.1
The Real Number System

http://mathworld.wolfram.com/Polynomial.html

At home, on your own,

Google (etc.) the word:

polynomial

Or these others:

algebraic expression,

or factoring

Algebra

From Wikipedia:

derived from an Arabic word **Al-Jabr**
in the title of a treatise written in 820
by the Persian mathematician,
Muhammad bin Musa al-Khwarizmi

Let’s take a look at some math theory…

Sets

Given that **A = { 4, 5, 6 }** establishes a set of the three numbers 4, 5, and 6;

4 ∈ A indicates *4* ***is*** *in the set A*

8 ∉ A indicates *8 is* ***not*** *in the set A*

Similarly, **B = {Exxon, Ford, Sony}** yields

Ford ∈ B indicating *Ford* ***is*** *in the set B*

8 ∉ B indicating *8 is* ***not*** *in the set B*

Rational & Irrational Numbers

- **Rational Numbers** are a ratio of two integers:

  p/q where q ≠ 0

  *As **q = 1 is rational, all integers are rational***

  1/2 = 0.500000 ... *2/11 = 0.181818 ...*

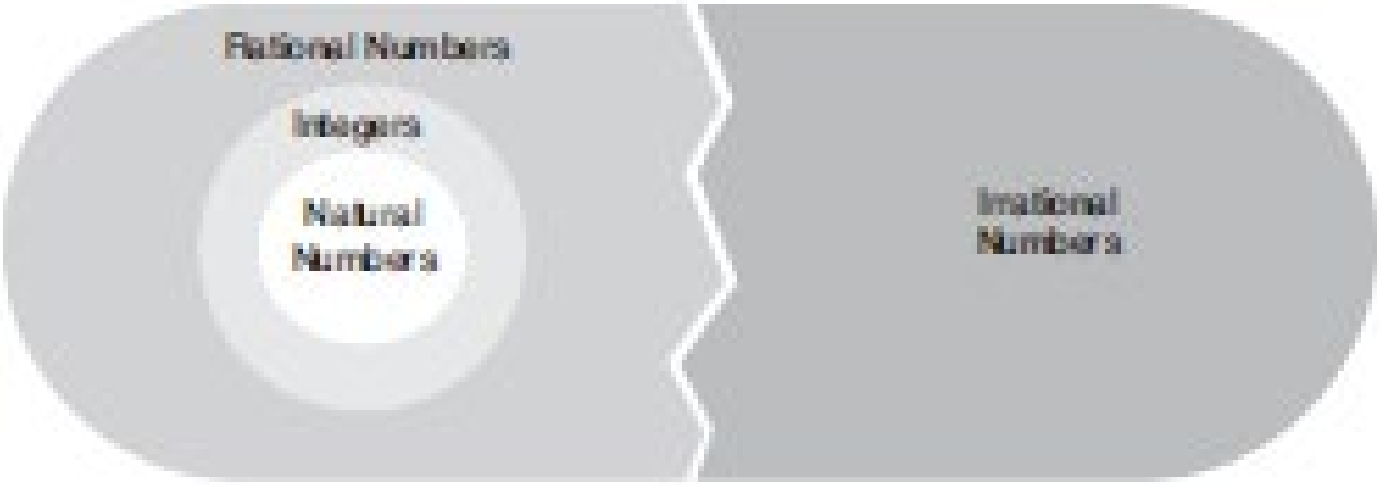
  where the digit patterns repeat.

- **Irrational Numbers** – *NOT* a ratio of integers:

  $\pi = 3.141592654$ $\sqrt{2} = 1.414213562$

- **Real Numbers** = *The set of all Rational and Irrational Numbers*

Real Numbers {Rational, Irrational Numbers}



- **Rational Numbers: {Natural Numbers, Integers}**
  - **Natural Numbers**: 1, 2, 3, → Integers
  - The concept of '**Nothing**' = 0
  - **Negative Numbers**: -1, -2, -3, etc.
  - The set of **Integers**: -2, -1, 0, 1, 2, 3, etc.
- **Irrational Numbers; decimal numbers never repeating** (π)

Graphing Calculator Alert

- **Rational Numbers**: *Calculators display only a finite number of digits, therefore they may be **susceptible to rounding errors**...*

  (1/3 = 0.3333333333) → Press: [1][÷][3][=]

- **Irrational Numbers**: *Calculators display only a finite number of digits, therefore they provide a **rational decimal approximation** for any irrational numbers.*

(√2 = 1.414213562) → Press [√][(][2][)][)][=]

Questions?

About Real Numbers

- Properties of Real Numbers
- Properties of Equality
- Additional properties

Properties of Real Numbers - 1

Example	Algebraic Expression	Property
$3 + 4$ is a real number.	$a + b$ is a real number.	**Closure under addition** The sum of two real numbers is a real number.
$2 \cdot 5$ is a real number.	$a \cdot b$ is a real number.	**Closure under multiplication** The product of two real numbers is a real number.
$4 + 8 = 8 + 4$	$a + b = b + a$	**Commutative under addition** We may add real numbers in any order.
$3(5) = 5(3)$	$a(b) = b(a)$	**Commutative under multiplication** We may multiply real numbers in any order.
$(2 + 5) + 3 = 2 + (5 + 3)$	$(a + b) + c = a + (b + c)$	**Associative under addition** We may group the addition of real numbers in any order.
$(2 \cdot 5)3 = 2(5 \cdot 3)$	$(ab)c = a(bc)$	**Associative under multiplication** We may group the multiplication of real numbers in any order.
$4 + 0 = 4$	$a + 0 = a$	**Additive identity** The sum of the unique real number 0 and any real number leaves that number unchanged.

Properties of Real Numbers - 2

Example	Algebraic Expression	Property
$3(1) = 3$	$a(1) = a$	**Multiplicative identity** The product of the unique real number 1 and any real number leaves that number unchanged.
$5 + (-5) = 0$	$a + (-a) = 0$	**Additive inverse** The number $-a$ is called the **negative**, **opposite**, or **additive inverse** of a. If $-a$ is added to a, the result is the additive identity 0.
$7\left(\frac{1}{7}\right) = 1$	If $a \neq 0$, $a\left(\frac{1}{a}\right) = 1$	**Multiplicative inverse** The number $\frac{1}{a}$ is called the **reciprocal**, or **multiplicative inverse**, of a. If $\frac{1}{a}$ is multiplied by a, the result is the multiplicative identity 1.
$2(5 + 3) = (2 \cdot 5) + (2 \cdot 3)$ $(4 + 7)2 = (4 \cdot 2) + (7 \cdot 2)$	$a(b + c) = ab + ac$ $(a + b)c = ac + bc$	**Distributive laws** If one number multiplies the sum of two numbers, we may add the two numbers first and then perform the multiplication; or we may multiply each pair and then add the two products.

Properties of Equality

Example	Algebraic Expression	Property
$3 = 3$	$a = a$	Reflexive property
If $\frac{6}{3} = 2$ then $2 = \frac{6}{3}$.	If $a = b$ then $b = a$.	Symmetric property
If $\frac{6}{3} = 2$ and $2 = \frac{8}{4}$, then $\frac{6}{3} = \frac{8}{4}$	If $a = b$ and $b = c$, then $a = c$.	Transitive property
If $\frac{6}{3} = 2$, then we may replace $\frac{6}{3}$ by 2 or we may replace 2 by $\frac{6}{3}$.	If $a = b$, then we may replace a by b or we may replace b by a.	Substitution property

*Ten (10) Homework Assignment Quizzes (worth 2.5% each) for a total of **25%** of your final grade…*

PROOF:

10x = 25; 25/10 = x; 2.5 = x

Additional Properties

Example	Algebraic Expression	Property
If $\frac{6}{3} = 2$ then $\frac{6}{3} + 4 = 2 + 4$ $\frac{6}{3}(5) = 2(5)$	If $a = b$, then $a + c = b + c$ $ac = bc$	The same number may be added to both sides of an equation. Both sides of an equation may be multiplied by the same number.
If $\frac{6}{3} + 4 = 2 + 4$ then $\frac{6}{3} = 2$.	If $a + c = b + c$ then $a = b$.	Cancellation law of addition
If $\frac{6}{3}(5) = 2(5)$ then $\frac{6}{3} = 2$.	If $ac = bc$ with $c \neq 0$ then $a = b$.	Cancellation law of multiplication
$2(0) = 0(2) = 0$ $2(3) = 0$ is impossible.	$a(0) = 0(a) = 0$ If $ab = 0$ then $a = 0$ or $b = 0$.	The product of two real numbers can be zero only if one of them is zero. The real numbers a and b are said to be factors of the product ab.
$-(-3) = 3$ $(-2)(3) = (2)(-3) = -6$ $(-1)(3) = -3$ $(-2)(-3) = 6$ $(-2) + (-3) = -(2 + 3) = -5$	$-(-a) = a$ $(-a)(b) = (a)(-b) = -(ab)$ $(-1)(a) = -a$ $(-a)(-b) = ab$ $(-a) + (-b) = -(a + b)$	Rules of signs

Algebra *is all about solving problems…*

- Given a set of plans: How much lumber, paint, etc, will be used to build something.
- Combined with physics: Calculation of stresses and strains in building elements; deflections of beams, plates, etc.
- Creating computer graphics: algebra & trigonometry are needed to rotate a perspective rendition of an scene on the screen. Every line must be recalculated and hidden portions of lines must be found, erased.
- With calculus (a continuation of algebra) used throughout science, engineering, and the financial industry…

Algebra is *symbolic math*

Q1. A car travels 2 hours in the morning and then another 3 hours in the afternoon.

How long was the trip?

Q2. You have 2 apples in your cart and put another 3 in it. How many apples do you have?

Ans: Both problems can be represented as:

$$\mathbf{2x + 3x = 5x}$$

where x is a *symbol* rather than an *actual unit*.

Equations are "sentences"

The numerical value of an expression on the left side of the equal sign is equal to the numerical value of the expression on the right side.

Examples : $x + 3 = 7$

x must have the value 4 to satisfy this equation containing one unknown.

$$2x + 3y = 10$$

Many different **x,y pairs can satisfy equations containing two unknowns**

Main Goal of this Course

Learn to solve practical problems

by

creating

and then solving

an appropriate algebraic equation.

Questions?

In Class Assignments

- Work in teams of **two**
- You do the first problem; your partner checks it; your partner does the next and so on

In small classes, you will work individually…

- Ask questions if you get stuck
- Sometimes, but not always, there are several correct methods to arrive at the correct answer.

One method that is NOT is:

Looking up the answer in the answer key…

Quick Problems

Version 1.

You bought a new car after the dealer lowered the original price by 17%. You paid $21,000.

Calculate the original price.

You have 10 minutes to solve this problem

Quick Problems - Ans

Version 1.

... dealer lowered the original price by 17% , You paid $21,000.

Calculate the original price.

x (1 - 0.17) = 21000

21000÷(1 - 0.17) = x

x = $25,301.20

Quick Problems

Version 2.

You bought a new car on sale. You paid $21,000 - which was 83% of the original price. Calculate the original price.

You have 5 minutes to solve this problem

Quick Problems - Ans

Version 2.

… paid \$21,000 = 83% of the original price.
Calculate the original price.

$$0.83x = 21000$$

$$x = 21000 \div 0.83$$

$$x = \$25{,}301.20$$

(This is just another way to state this problem…)

Quick Problems

Question: On sale for 80% of its original price, a car cost $19,000. What was the original price?

You have 5 minutes to solve this problem

Quick Problems - Ans

… 80% of its original price = $19,000. What was the original price?

Answer: Let *P represent the original price, the "unknown".*

We are given that 80% of P is 19000.

We can write this immediately as an algebraic equation ("sentence"), ***0.8 P =19000***

Solve for P by dividing each side of the equation by 0.8, so:

$$P = 19000 \div 0.8 = 23750$$

Quick Problems

Question: A car is on sale for 80% of its original price of $20,000. What is the sale price?

You have 5 minutes to solve this problem

Quick Problems - ANS

…on sale for 80% of $20,000.

What is the selling price?

P = 20,000 (0.80)

P = $16,000

Quick Problems

- If the width of a rectangle is reduced by 10% and the height is increased by 10%, how much is the new area in terms of the old area?

You have 10 minutes to solve this problem

Quick Problems - ANS

- *If the width of a rectangle is reduced by 10% and the height is increased by 10%, how much is the new area in terms of the old area?*

$\mathbf{A_0 = w \bullet h;}$ $\qquad$ $\mathbf{A_1 = [(1\text{-}.1)(w)]\ [(1\text{+}.1)(h)]}$

$$A_1 = (0.9)(1.1)wh = 0.99wh$$

$$A_1 / A_0 = [0.99(wh)]/(wh) = 99\%$$

Quick Problems

- Suppose we have a 20 ft rope that we want to cut into 3/4 ft pieces. How many pieces will this rope yield? If there is one, how long is the fractional piece?

You have 10 minutes to solve this problem

Quick Problems - ANS

… a **20** ft rope (is) cut into **3/4** ft pieces.

How many pieces will this rope yield?

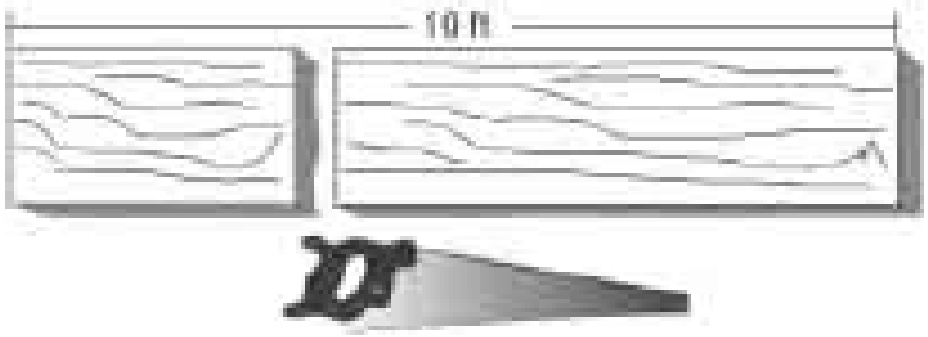
20/(3/4) = 26 2/3 pieces

… How long is the fractional piece?

(2/3)(3/4) = (2•3/3•4) = 6/12 = **½ foot long**

Chapter 1 - Page 12, Problem 59

59. A board 10 feet long is cut into two pieces, the lengths of which are in the ratio of 2:3. Find the lengths of the pieces.



You have 5 minutes to solve this problem

Ch1, Pg 12, Problem 59 - Ans

59. A board 10 feet long is cut into two pieces, the lengths of which are in the ratio of 2:3. Find the lengths of the pieces.

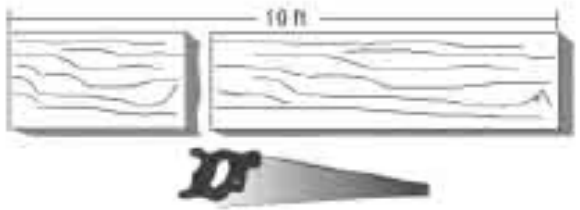
Ans: *Ratio of 2:3 means the Boards are 2x, 3x long*

Equations: $2x + 3x = 10;\ 5x = 10;\ x = 2$

Smaller Board: 2(2 feet long) = 4 feet long

Larger Board: 3(2 feet long) = 6 feet long

Final Answer: 4 feet long and 6 feet long*



(*) Note that units <u>MUST</u> always be included in your final answer…

(A ratio of 1:2 would yield 3.33 ft; 6.67 ft)

Chapter 1 - Page 12, Problem 60

60. An alloy is 3/8 copper, 5/12 zinc, and the balance lead. How much lead is there in 282 pounds of alloy?

Ch1, Pg12, Problem 60 - Ans

60. An alloy is 3/8 copper, 5/12 zinc, and the balance lead. How much lead is there in 282 pounds of alloy?

Ans. Equation is: $(3/8 + 5/12) + x = 1$

a. $3/8 \bullet 3 \rightarrow 9/24$ and $5/12 \bullet 2 \rightarrow 10/24$

thus, $19/24 + x = 1$ and $x_{lead} = 5/24 = 0.208$*

b. $282 \text{ lbs} \bullet 0.208 = 58.656$ lbs lead in the alloy

***Final Answer**: 58.656 pounds*

**Using a calculator to find decimals and not using conversion to the lowest common denominator:*

$(0.375 + 0.417) = 0.792$ and thus, again $x_{lead} = 0.208$

NOTE:

We will review this problem and the challenges of rounding errors next class…

Questions?

Homework Assignment Set #1

Section 1.1 (The Real Number System)

- **pages 10-11 (page 9 in 5th Ed):**

  Problems 9 through 18

  Problem 19 (*Hint: If you are stuck, Google on "sum of two irrational"*)

  Problems 20, 23, 24, 28, 35

  Problem 38 (***Also find values for a and b for which the statement is true***.)

  Problems 40, 53, 54, 55, 57, 58, 61, 62

  NOTE: *The online homework pdf has the full actual problems written out…*

Due Next Classes:

NOTE: Next Monday is Labor Day

Pratt Holiday! No Classes!

In Session 2:

- ***DUE:*** ***Textbook readings***
- ***DUE:*** ***Homework Set #01***
- ***REVIEW:*** ***Homework Set #01***

Due – Session 3:

- ***Nomenclature, Notes***
- ***Lecture and Problem Review***
- ***Due Homework Set #02***
- ***REVIEW:*** ***Homework Set #02***
- ***QUIZ:*** ***Homework Set #01***

Any Questions?

Send me an email …

crubenst@pratt.edu

or

c.rubenstein@ieee.org

End