Pratt

Math 150 – Fall 2020
Algebra & Trigonometry

Charles Rubenstein, Ph. D.
Professor of Engineering & Information Science

Session 1: Monday 8/24/19
6:30pm - 9:20pm
via REMOTE LEARNING
Revision 1

Instructor Contact Information

Dr. Charles Rubenstein <crubenst@pratt.edu>
Professor of Engineering & Information Science
Faculty Office: ARC G-49

Fall 2020 Virtual Office hours *ONLY*
Wednesdays 10:00am-2:00pm via Zoom Meeting
To make your appointment
Send me an email at least one day in advance :
crubenst@pratt.edu

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For Today's Class:

Session 1: Posted Online on Wednesday 19 August 2020
- ***Download & Review: Syllabus and Course Schedule***
- ***Textbook and Calculator needs***
- ***Review of the Class Archive Concept (Homework online)***
- ***Remote Learning via Lecture and in-class problem solving***
- ***Download & Review: Textbook Chapter 1***

For our next class – Session 2:
- **Session 2: Posted Online on Wednesday 26 August 2020**
- ***Readings, homework***
- ***DUE: Homework Set #01 by 12:00Noon Monday 31 August!***
- ***Homework Review Set #01 & Quiz #01 during class***

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Math 150

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Syllabus Review:
Math150 - Algebra & Trigonometry

Who is *Dr. Rubenstein ?*

- Subject Background in
  - Bioengineering
  - Electrical Engineering
  - Systems Analysis
  - Information Science
- Certifications
  - Microsoft Trainer
  - CompTIA A+ Certified
- Professional Society Memberships
  - **ALISE** (*Member*), **IET** (*Fellow, Chartered Engineer*)
  - **IEEE** (*Senior Member*) *Member of the 2010-2011 Board of Directors of this 430,000 member professional engineering organization that produces more than one third of the world's electrotechnology information*

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But… It's really all about …You!

Your Pratt Education …
is founded on choices of

- **Career**
- **Topics of Study**
- **Background**

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My Style of Teaching

Each class will consist of a mix of

- *Prepared PowerPoint Slides posted to the class website a week prior to each Zoom Class meeting (asynchronous)*
- *A Zoom Class Lecture/Discussion on the week's lesson (synchronous)*
  - *Review of Homework Assignments*
  - *Interactive Lectures* = **Class participation**

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Math 150 – Class Topics

1. **The Foundations of Algebra**
2. **Equations and Inequalities**
3. **Functions**
4. **Polynomial Functions**
5. **Rational Functions and Conic Sections**
6. **Exponential and Logarithmic Functions**
7. **The Trigonometric Functions**
8. **Analytic Trigonometry**
9. **Applications of Trigonometry**
10. **Systems of Equations and Inequalities**
11. **Matrices, Linear Systems, and Determinants**
12. **Topics in Algebra**

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Draft Schedule: Math 150 – Fall 2020 – Remote Learning

Monday	Notes
24-Aug	1. Introduction: Numbers, Arithmetic Operations, Fractions
31-Aug	2. Manipulation of Algebraic Expressions; *Hwk #1 Due @ Noon*
7-Sep	NO CLASSES – Labor Day
14-Sep	3. Solving Linear and Quadratic Equations of One Variable; *Hwk #2 Due*
21-Sep	4. Solving Equations of Two Variables; *Hwk #3 Due*
28-Sep	NO CLASSES – Instructor Holiday
5-Oct	5. Creating Equations – Polynomial Functions; *Hwk #4 Due; Exam #1 Emailed*
12-Oct	6. Polynomial Functions, continued; *Exam #1, Hwk #5 Due at 12:00pm Noon*
19-Oct	7. Functions, Graphing, Exponents and Logarithms; *Exam Review; Hwk #6*
26-Oct	8. Trigonometric Functions, Pythagorean Theorem; *Hwk #7 Due*
2-Nov	9. Applications of Trigonometry; *Hwk #8 Due*
9-Nov	10. Analytic Trigonometry: Identities & Graphing; *Exam #2 Emailed; Hwk #9*
16-Nov	11. Areas and Volumes of Geometric Solids; *Exam #2, Hwk #10 Due at Noon*
23-Nov	12. Systems of Equations and Inequalities; *Exam #2 Review*
30-Nov	13. Series and Sequences, Review topics; *Final Exam Emailed*
7-Dec	Final Examination *Due at 12:00pm Noon*

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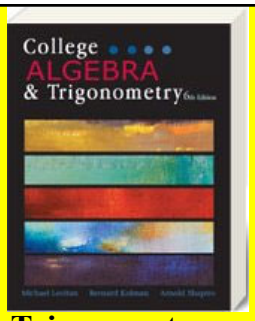
Recommended Textbook:



Pub Date: 2011
Publisher: BVT Publishing
ISBN: 978-1-60229-880-4
($45.00)

College Algebra & Trigonometry

Sixth Edition

Michael Levitan, Bernard Kolman, and Arnold Shapiro

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Recommended: *TI-83 Plus or ANY Graphing Calculator (hp, TI, etc.)*

Any graphing calculator may be used in this class

Mr. Ramus (CM/FM – PMC; pramus@pratt.edu) has loaner units if you do not wish to purchase one

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Your Grade in Math 150

Homework Quizzes (30%)
Two Exams (20% each)
Final Exam (30%)

Your Class Participation… (priceless!)

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About Homework Assignments

Homework is an essential part of this class.
Doing the assignments will allow you to keep up with the class and your studies.

Homework must be emailed to me
by 12:00Noon on each day of classes.

There are TWELVE (12) Homework assignments.
For the first ten (10) Homework assignments I will select three (3) problems from each assignment to grade. 1% per correct answer, maximum of 3% per homework, for a total of **30%** of your final grade.

There will be NO make up 'quizzes'
as we will review the homework problems in class

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About Your Exams

- There will be two (2) take home exams worth 20% each = **40%**
  (one to two-hour take home exams - to make up for class schedule challenges)
- There will be a take home FINAL exam worth **30%**

These are Open Book exams
A Formula Sheet will be provided, *no need to memorize…*

Examinations are designed to see what you have learned this semester as well as
to see what you DIDN'T learn
and prepare you for your Calculus class

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Class Session Archives

All of our course materials will be found on the Class Website at:

www.CharlesRubenstein.com/150

The next slide indicates the files currently on the website

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www.CharlesRubenstein.com/150

- **syllabus.pdf** = **Syllabus** ***with DRAFT Class Schedule***
- **150_Zoom_Info.pdf**
  ZoomTipsForStudents.pdf
- **Levitan6ed_ch1.pdf**
  = ***6th Edition Textbook's First Chapter***
- **HWK1to6.pdf** = ***Homework Sets #01-#06***
- **FormulaSheet.pdf**

Also there: **20fa01.pdf** = *This slide set**
20fa01_h.pdf = *slides as 6-up handouts**
**Available by Noon Wednesday 1 week before Class Meetings*

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Math 150

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Chapter 1 – Part 1
Foundations of Algebra

On Memorization of Formulas

You are NOT required to memorize anything.

When we do the proof of the Pythagorean Theorem you should want to memorize it unless you are already able to write down an equivalent proof.

All exams will be open book, open notes.

You are responsible for writing all formulas and definitions (sine, cosine, and tangent) on a ***SINGLE sheet*** in your notebook if you do not have them memorized.

After seeing these formulas enough times, you'll quite likely unconsciously memorize them.

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Chapter 1.1

The Real Number System

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http://mathworld.wolfram.com/Polynomial.html

At home, on your own,
Google (etc.) the word:
polynomial
Or these others:
algebraic expression,
or factoring

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Algebra

From Wikipedia: Algebra
is derived from an Arabic word **Al-Jabr**
in the title of a treatise written in 820
by the Persian mathematician,
Muhammad bin Musa al-Khwarizmi

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Let's look at some math theory…

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Sets

Curly Braces **{ }** ***enclose the set contents***

The symbol **∈** ***indicates the element IS in the set***

The symbol **∉** ***indicates the element IS NOT in the set***

Examples

A = { 4, 5, 6 } establishes a set of the three numbers:
4, 5, and 6 where
4 ∈ A indicates *4* ***is*** *in the set* ***A***
8 ∉ A indicates *8* ***is not*** *in the set* ***A***

Similarly, **B = {Exxon, Ford, Sony}** yields
Ford ∈ B indicating *Ford* ***is*** *in the set* ***B***
8 ∉ B indicating *8* ***is not*** *in the set* ***B***

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Rational & Irrational Numbers

- **Rational Numbers** are a ratio of two integers:

  p/q where q ≠ 0

*As **q = 1 is rational, all integers are rational***

1/2 = 0.500000 … 2/11 = 0.181818 …

where the digit patterns repeat.

- **Irrational Numbers** – are *NOT* a ratio of integers:

  π = 3.141592654 √2 = 1.414213562

- **Real Numbers** = *The set of all Rational and Irrational Numbers*

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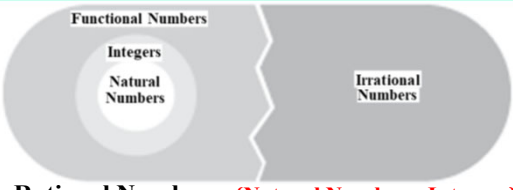
Real Numbers {Rational, Irrational Numbers}



- **Rational Numbers: {Natural Numbers, Integers}**
  - **Natural Numbers**: 1, 2, 3, → Integers
  - The concept of '**Nothing**' = 0
  - **Negative Numbers**: -1, -2, -3, etc.
  - The set of **Integers**: -2, -1, 0, 1, 2, 3, etc.
- **Irrational Numbers; decimal numbers never repeating** (π)

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Graphing Calculator Alert

Rational Numbers:

*Calculators display only a finite number of digits, therefore they may be **susceptible to rounding errors**…*

(1/3 = 0.3333333333) → Press: [1][÷][3][=]

NOTE: On TI Calculators [ENTER] is "[=]"

Irrational Numbers:

*Calculators display only a finite number of digits, therefore they provide a **rational decimal approximation** for any irrational numbers.*

(√2 = 1.414213562) → Press: [√][2][ )][=]

Please note that √ is actual "[2ND][x²]" and it provides first "("

In this simple case, forgetting the closing ")" before [ENTER] is OK

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About Real Numbers

- Properties of Real Numbers
- Properties of Equality
- Additional properties

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Properties of Real Numbers - 1

Example	Algebraic Expression	Property
3 + 4 is a real number.	$a + b$ is a real number.	**Closure under addition** The sum of two real numbers is a real number.
2 · 5 is a real number.	$a \cdot b$ is a real number.	**Closure under multiplication** The product of two real numbers is a real number.
4 + 8 = 8 + 4	$a + b = b + a$	**Commutative under addition** We may add real numbers in any order.
3(5) = 5(3)	$a(b) = b(a)$	**Commutative under multiplication** We may multiply real numbers in any order.
(2 + 5) + 3 = 2 + (5 + 3)	$(a + b) + c = a + (b + c)$	**Associative under addition** We may group the addition of real numbers in any order.
(2 · 5)3 = 2(5 · 3)	$(ab)c = a(bc)$	**Associative under multiplication** We may group the multiplication of real numbers in any order.
4 + 0 = 4	$a + 0 = a$	**Additive identity** The sum of the unique real number 0 and any real number leaves that number unchanged.

Table 1, Page 6

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Properties of Real Numbers - 2

Example	Algebraic Expression	Property
$5(1) = 5$	$a(1) = a$	**Multiplicative identity** The product of the unique real number 1 and any real number leaves that number unchanged.
$5 + (-5) = 0$	$a + (-a) = 0$	**Additive inverse** The number $-a$ is called the **negative, opposite,** or **additive inverse** of a. If $-a$ is added to a, the
$7\left(\frac{1}{7}\right) = 1$	If $a \neq 0$, $a\left(\frac{1}{a}\right) = 1$	**Multiplicative inverse** The number $\frac{1}{a}$ is called the **reciprocal,** or **multiplicative inverse,** of a. If $\frac{1}{a}$ is multiplied by a, the result is the multiplicative identity 1.
$2(5 + 3) = (2 \cdot 5) + (2 \cdot 3)$ $(4 + 7)2 = (4 \cdot 2) + (7 \cdot 2)$	$a(b + c) = ab + ac$ $(a + b)c = ac + bc$	**Distributive laws** If one number multiplies the sum of two numbers, we may add the two numbers first and then perform the multiplication; or we may multiply each pair and then add the two products.

Table 1, Page 6-7

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Properties of Equality

Example	Algebraic Expression	Property
$3 = 3$	$a = a$	Reflexive property
If $\frac{6}{3} = 2$ then $2 = \frac{6}{3}$.	If $a = b$ then $b = a$.	Symmetric property
If $\frac{6}{3} = 2$ and $2 = \frac{8}{4}$, then $\frac{6}{3} = \frac{8}{4}$	If $a = b$ and $b = c$, then $a = c$.	Transitive property
If $\frac{6}{3} = 2$, then we may replace $\frac{6}{3}$ by 2 or we may replace 2 by $\frac{6}{3}$.	If $a = b$, then we may replace a by b or we may replace b by a.	Substitution property

Here's an example…
Ten (10) Homework Assignment Quizzes (worth 3% each) for a total of ***30%*** *of your final grade…*

PROOF:

10x = 30; 30/10 = x; 3 = x

Table 2, Page 7

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Additional Properties

Example	Algebraic Expression	Property
If $\frac{6}{3} = 2$ then $\frac{6}{3} + 4 = 2 + 4$ $\frac{6}{3}(5) = 2(5)$	If $a = b$, then $a + c = b + c$ $ac = bc$	The same number may be added to both sides of an equation. Both sides of an equation may be multiplied by the same number.
If $\frac{6}{3} + 4 = 2 + 4$ then $\frac{6}{3} = 2$.	If $a + c = b + c$ then $a = b$.	Cancellation law of addition
If $\frac{6}{3}(5) = 2(5)$ then $\frac{6}{3} = 2$.	If $ac = bc$ with $c \neq 0$ then $a = b$.	Cancellation law of multiplication
$2(0) = 0(2) = 0$ $2(3) = 0$ is impossible.	$a(0) = 0(a) = 0$ If $ab = 0$ then $a = 0$ or $b = 0$.	The product of two real numbers can be zero only if one of them is zero. The real numbers a and b are said to be **factors** of the product ab.
$-(-3) = 3$	$-(-a) = a$	Rules of signs
$(-2)(3) = (2)(-3) = -6$	$(-a)(b) = (a)(-b) = -(ab)$	
$(-1)(3) = -3$	$(-1)(a) = -a$	
$(-2)(-3) = 6$	$(-a)(-b) = ab$	
$(-2) + (-3) = -(2 + 3) = -5$	$(-a) + (-b) = -(a + b)$	

Table 3, Page 8

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Algebra *is all about solving problems…*

- Given a set of plans: How much lumber, paint, etc, will be used to build something.
- Combined with physics: Calculation of stresses and strains in building elements; deflections of beams, plates, etc.
- Creating computer graphics: algebra & trigonometry are needed to rotate a perspective rendition of an scene on the screen. Every line must be recalculated and hidden portions of lines must be found, erased.
- With calculus (a continuation of algebra) used throughout science, engineering, and the financial industry…

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Algebra is ***symbolic math***

Q1. A car travels 2 hours in the morning and then another 3 hours in the afternoon.

How long was the trip?

Q2. You have 2 apples in your cart and put another 3 in it. How many apples do you have?

Ans: Both problems can be represented symbolically as:

2x + 3x = 5x

where x is a *symbol* rather than an *actual unit*.

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Equations are "sentences"

The numerical value of an expression on the left side of the equal sign is equal to the numerical value of the expression on the right side.

Examples :

For ***x + 3 = 7***

x must have the value 4 to satisfy this equation containing one unknown.

BUT, for ***2x + 3y = 10***

Many different **x,y pairs can satisfy equations containing two unknowns**

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Main Goal of this Course

Learn to solve practical problems

by

creating

and then solving

an appropriate algebraic equation.

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Quick Problems

Question 1 ***(version 1)***.

You bought a new car after the dealer lowered the original price by 17%. You paid $21,000.

Calculate the original price.

You have 10 minutes to solve this problem

(The results are on the next slide if you get stuck…)

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Quick Problems – Ans Q1.1

Question 1 ***(version 1)***.

… dealer lowered the original price by 17% , You paid $21,000.

Calculate the original price.

$$x\,(1 - 0.17) = 21000$$

$$21000 \div (1 - 0.17) = x$$

x = $25,301.20

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Quick Problems

Question 1 ***(version 2)***.

You bought a new car on sale. You paid $21,000 - which was 83% of the original price. Calculate the original price.

You have 5 minutes to solve this problem

(The results are on the next slide if you get stuck…)

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Quick Problems – Ans Q1.2

Question 1 ***(version 2)***.

… paid $21,000 = 83% of the original price. Calculate the original price.

$$0.83x = 21000$$

$$x = 21000 \div 0.83$$

x = $25,301.20

(This is just another way to state this problem…)

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Quick Problems

Question 2:

On sale for 80% of its original price, a car cost $19,000. What was the original price?

You have 5 minutes to solve this problem

(The results are on the next slide if you get stuck…)

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Quick Problems – Ans Q2

… 80% of its original price = $19,000. What was the original price?

Answer: Let *P represent the original price, the "unknown".*

We are given that 80% of P is 19000.

We can write this immediately as an algebraic equation ("sentence"), ***0.8 P =19000***

Solve for P by dividing each side of the equation by 0.8, so:

$$P = 19000 \div 0.8 = 23750$$

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Quick Problems

Question 3:

A car is on sale for 80% of its original price of $20,000. What is the sale price?

You have 5 minutes to solve this problem

(The results are on the next slide if you get stuck…)

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Quick Problems – ANS Q3

…on sale for 80% of $20,000.

What is the selling price?

P = 20,000 (0.80)

P = $16,000

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Quick Problems

Question 4:

If the width of a rectangle is reduced by 10% and the height is increased by 10%, how much is the new area in terms of the old area?

You have 10 minutes to solve this

(The results are on the next slide if you get stuck…)

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Quick Problems – ANS Q4

If the width of a rectangle is reduced by 10% and the height is increased by 10%, how much is the new area in terms of the old area?

$A_0 = w \cdot h$; **$A_1 = (1-.1)(w)(1+.1)(h)$**

Therefore, in our general rectangle;

$A_1 = (0.9)(1.1)\mathbf{(w)(h)} = 0.99\ \mathbf{wh}$

$A_1 / A_0 = [0.99wh]/[wh] = \mathbf{99\%}$

NOTE: *in a square,* ***w = h = x****, but answer is same*

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Quick Problems

Question 5:

Suppose we have a 20 ft rope that we want to cut into 3/4 ft pieces. How many pieces will this rope yield? If there is one, how long is the fractional piece?

You have 10 minutes to solve this

(The results are on the next slide if you get stuck…)

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Quick Problems – ANS Q5

… a **20** ft rope (is) cut into **3/4** ft pieces.

How many pieces will this rope yield?

20/(3/4) = 26 2/3 pieces

… How long is the fractional piece?

(2/3)(3/4) = (2•3/3•4) = 6/12 = **½ foot long**

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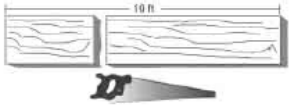
Chapter 1 - Page 12, Problem 59

59. A board 10 feet long is cut into two pieces, the lengths of which are in the ratio of 2:3. Find the lengths of the pieces.

You have ten minutes to solve this



(The results are on the next slide if you get stuck…)

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Ch1, Pg 12, Problem 59 - Ans

59. A board 10 feet long is cut into two pieces, the lengths of which are in the ratio of 2:3. Find the lengths of the pieces.

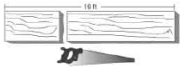
Ans: *Ratio of 2:3 means the Boards are 2x, 3x long*

Equations: 2x + 3x = 10; 5x = 10; x = 2

Smaller Board: 2(2 feet long) = 4 feet long

Larger Board: 3(2 feet long) = 6 feet long

Final Answer: 4 feet long and 6 feet long*



*(*) Note that units **MUST** always be included in your final answer…*

(A ratio of 1:2 would yield 3.33 ft; 6.67 ft)

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Chapter 1 - Page 12, Problem 60

60. An alloy is 3/8 copper, 5/12 zinc, and the balance lead. How much lead is there in 282 pounds of alloy?

You have ten minutes to solve this

(The results are on the next slide if you get stuck…)

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Ch1, Pg12, Problem 60 - Ans

60. An alloy is 3/8 copper, 5/12 zinc, and the balance lead. How much lead is there in 282 pounds of alloy?

Ans. Equation is: (3/8 + 5/12) + x = 1

a. 3/8 • 3 → 9/24 and 5/12 • 2 → 10/24

thus, 19/24 + x = 1 and x_{lead} = 5/24 = 0.208*

b. 282 lbs • 0.208 = 58.656 lbs lead in the alloy

Final Answer: ***58.656 pounds of lead***

**Using a calculator to find decimals and not using conversion to the lowest common denominator: (0.375 + 0.417) = 0.792 and thus, again x_{lead} = 0.208 lbs*

DON'T FORGET THE UNITS!

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Homework Assignment Set #1

Section 1.1 (The Real Number System)

pages 10-11:

Problems 9 through 18

Problem 19 (*Hint: If you are stuck, Google on "sum of two irrational"*)

Problems 20, 23, 24, 28, 35

Problem 38 (***Also find values for a and b for which the statement is true***.)

Problems 40, 53, 54, 55, 57, 58, 61, 62

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Due Next Class:

***Due* – Session 2:**

- ***DUE:* Textbook readings**
- ***Lecture and Problem Review***
- ***Homework Set #01:* Due by 12:00Noon**
- ***'Quiz' on Homework Set #01***

***In class* – Session 3:**

- ***Nomenclature, Notes***
- ***Lecture and Problem Review***
- ***Homework Set #02:* Due by 12:00Noon**
- ***'Quiz' on Homework Set #02***

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Any Questions?

Send me an email …

crubenst@pratt.edu

or

c.rubenstein@ieee.org

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End

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