Pratt

Math 150 – Fall 2020

Algebra & Trigonometry

Charles Rubenstein, Ph. D.
Professor of Engineering & Information Science

Session 2: Monday 8/31/20

6:30pm - 9:20pm

via REMOTE LEARNING

Revision 1

Instructor Contact Information

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Faculty Office: ARC G-49

Fall 2020 Virtual Office hours *ONLY*

Wednesdays 10:00am-2:00pm via Zoom Meeting

To make your appointment

Send me an email at least one day in advance :

crubenst@pratt.edu

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This Class Session

Class Sessions Posted Online Friday before Class

- ***DUE:* Homework Set #01 by 12:00Noon Monday 31 August!**
- ***NOTE: Quiz 1 = four problems from hwk***
- ***Due:* Textbook readings**
- ***Lecture: Manipulation of Algebraic Expressions***
- ***Review:* Homework Set #01**

NO CLASSES Next Monday - 7 September = LABOR DAY

***In class* – Session 3:**

- ***Due & Review:* Textbook readings, Homework Set #02**
- ***Lecture: Solving Linear and Quadratic Equations of One Variable***

***In class* – Session 4:**

- ***Due & Review:* Textbook readings, Homework Set #03**
- ***Lecture: Solving Equations of Two Variables***

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About the Homework Quizzes

Since there are dozens of practice problems within each homework set, I feel it unnecessary to have you send ALL the problems to me each week.

I have selected four (4) problems from each homework for you to submit and - as long as at least three are answered correctly - receive 'quiz' credit of 3% for correct answers.

These are the selected problems for the first six homework assignments:

HWK #01: Section 1.1: 57, 58, 61, 62
HWK #02: Section 1.3: 49, 58, 82 and Section 1.4: 9
HWL #03: Review Section: 17 and Section 2.2: 1, 4, 12
HWK #04: Section 2.2: 25, 28, 48 and Section 2.3: 16
HWK #05: Section 2.4: 1, 4, 8, 21
HWK #06: Section 2.2: 54a, 54c, 55, 57

Homework 1 was due not later than Noon today.

The other homeworks are due each of the following weeks at Noon.

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Emailing me your Homework

As noted, I have selected four (4) problems from each homework for you to submit each week per the previous slide.

Homework is due not later than Noon class days.

HOW TO PREPARE YOUR ASSIGNMENT:

1. Use DARK pencil or pen.

If I can't read your work you get a ZERO!

2. It is preferable that you scan your work as a PDF and Save it as ***lastname_xx.pdf***

HOWEVER – IF YOU CAN NOT SCAN –

a. *Take a photo of your work*
b. *Insert the photo(s) into a Word document*
c. *Save as* ***lastname_xx.docx*** *or* ***lastname_xx.pdf***

Then email your file to me: ***c.rubenstein@ieee.org*** .

Email me ONLY the requested four (4) problems.
(Email any you might be challenged by in a separate document)

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Math 150 – Class Topics

1. **The Foundations of Algebra**
2. **Equations and Inequalities**
3. **Functions**
4. **Polynomial Functions**
5. **Rational Functions and Conic Sections**
6. **Exponential and Logarithmic Functions**
7. **The Trigonometric Functions**
8. **Analytic Trigonometry**
9. **Applications of Trigonometry**
10. **Systems of Equations and Inequalities**
11. **Matrices, Linear Systems, and Determinants**
12. **Topics in Algebra**

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Draft Schedule: Math 150 – Fall 2020 – Remote Learning

Monday	Notes
24-Aug	1. Introduction: Numbers, Arithmetic Operations, Fractions
31-Aug	2. Manipulation of Algebraic Expressions; *Hwk #1 Due @ Noon*
7-Sep	NO CLASSES – Labor Day
14-Sep	3. Solving Linear and Quadratic Equations of One Variable; *Hwk #2 Due*
21-Sep	4. Solving Equations of Two Variables; *Hwk #3 Due*
28-Sep	NO CLASSES – Instructor Holiday
5-Oct	5. Creating Equations – Polynomial Functions; *Hwk #4 Due; Exam #1 Emailed*
12-Oct	6. Polynomial Functions, continued; *Exam #1, Hwk #5 Due at 12:00pm Noon*
19-Oct	7. Functions, Graphing, Exponents and Logarithms; *Exam Review; Hwk #6*
26-Oct	8. Trigonometric Functions, Pythagorean Theorem; *Hwk #7 Due*
2-Nov	9. Applications of Trigonometry; *Hwk #8 Due*
9-Nov	10. Analytic Trigonometry: Identities & Graphing; *Exam #2 Emailed; Hwk #9*
16-Nov	11. Areas and Volumes of Geometric Solids; *Exam #2, Hwk #10 Due at Noon*
23-Nov	12. Systems of Equations and Inequalities; *Exam #2 Review*
30-Nov	13. Series and Sequences, Review topics; *Final Exam Emailed*
7-Dec	Final Examination *Due at 12:00pm Noon*

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www.CharlesRubenstein.com/150

- **HWK1to6.pdf** = ***Homework Sets #01-#06***
- **FormulaSheet.pdf**

Also there:

20fa02.pdf = *This slide set**

20fa02_h.pdf = *slides as 6-up handouts**

**My goal is to post these not later than Noon on Friday one week before our Zoom Class Meetings*

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Questions?

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Nomenclature

In today's class we will discuss:

- Algebraic Expressions
- Polynomial Expressions
- Arguments
- Equations
- Inequalities
- Radical Signs
- Absolute Values
- Magnitudes

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Chapter 1 – Part 2
The Foundations of Algebra

1.3 Algebraic Expressions & Polynomials
1.4 Factoring
1.5 Rational Expressions
1.6 Integer Exponents
1.7 Rational Exponents and Radicals

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Chapter 1 - Page 12, Problem 60

Revisiting…

Paying attention to rounding errors

60. An alloy is 3/8 copper, 5/12 zinc, and the balance lead. How much lead is there in 282 pounds of alloy?

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Ch1, Pg12, Problem 60 - Ans

60. An alloy is 3/8 copper, 5/12 zinc, and the balance lead. How much lead is there in 282 pounds of alloy?

Equation is: (3/8 + 5/12) + x = 1

a. 3/8 • 3 → 9/24 and 5/12 • 2 → 10/24

thus, 19/24 + x = 1 and x_{lead} = 5/24 = 0.208*

b. 282 lbs • 0.208 = 58.656 lbs lead in the alloy

***Final Answer**: 58.656 pounds*

**Using a calculator to find decimals and not using conversion to the lowest common denominator: (0.375 + 0.417) = 0.792 and thus, again x_{lead} = 0.208*

BUT is this the correct answer? See next slide…

Answers to 'n' decimal places

x_{lead} = 5/24 Using 5/24 = 0.2083333333333 …

We get 282 lbs • 0.2083333333333 … = **58.75 lbs**

Which is what we would get using fractions and NOT a calculator: (5/24) 282 = 235/4 = **58 ¾ lbs**.

*Using **0.208** we get **58.656** pounds, and using **0.21** we get **59.22** pounds; **THEREFORE***

As both the 3 decimal place and 2 decimal place answers are approximations,

UNLESS told to use a truncated amount, DON'T!

Homework Assignment Set #1

Section 1.1 (The Real Number System)

pages 10-11:

Problems 9 through 18

Problem 19 (*Hint: If you are stuck, Google on "sum of two irrational"*)

Problems 20, 23, 24, 28, 35

Problem 38 (***Also find values for a and b for which the statement is true***.)

Problems 40, 53, 54, 55, **57, 58, 61, 62**

Red problems used for homework quiz

Homework #1

We will go over the 'Quiz' problems in class and then they will appear in an updated slide set.

Let me know if there are any problems in this homework set that you might have had challenges on and we can review them…

It would be appreciated if you did so when you submitted your four homework problems for the 'quiz'

Homework Quiz #1 (57)

Section 1.1 (The Real Number System)

Page 11: Problems **57, 58, 61, 62**

57. On a map of Pennsylvania, 1 inch represents 10 miles. Find the distance represented by 3.5 inches.

Equation: **Inches/Miles = 1/10 = 3.5/x**

Cross multiply to get:

1x = 10 • 3.5 = 35 miles

(NOTE: YOU MUST INCLUDE UNITS IN YOUR ANSWER!)

Homework Quiz #1 (58)

Section 1.1 (The Real Number System)

Page 11: Problems **57, 58, 61, 62**

58. A car travels 135 miles on 6 gallons of gasoline. How far can it travel on 10 gallons of gasoline?

Equation: **Miles/Gallons = 135/6 = x/10**

Cross multiply to get:

135 • 10 = 6 • x

Divide both sides by 6:

x = 1350/6 = **225 miles**

Homework Quiz #1 (61)

Section 1.1 (The Real Number System)

Page 11: Problems **57, 58, 61, 62**

61. Which is the better value: 1 pound 3 ounces of beans for 85 cents, or, 13 ounces for 56 cents?

Convert to ounces:

1 pound, 3 ounces = 16 + 3 ounces = 19 ounces

Equations:

Price/Ounce = 85/19 = 4.47 cents/ounce

Price/Ounce = 56/13 = 4.31 cents/ounce

Thus: ***13 ounces for 56 cents = best buy.***

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Homework Quiz #1 (62)

Section 1.1 (The Real Number System)

Page 11: Problems **57, 58, 61, 62**

62. A piece of property is valued at $28,500.

What is the real estate tax at $75.30 per $1000.00 evaluation?

Equation: tax/value = $75.30 / $1000 = x / $28500

Cross multiply: ($75.30)($28,500) = $1000 x

21460.50 = $1000 x

$2146.05 = x, the real estate tax

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Questions?

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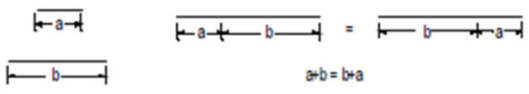
Some Laws, about Exponents Geometery and Perfect Triangles

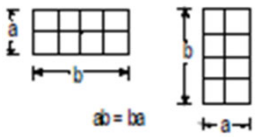
 22

"Laws" - 1



Commutative Law of Addition:

a + b = b + a (***NOTE: the order doesn't matter***)



Commutative Law of Multiplication:

ab = ba (**NOTE:** ***the order doesn't matter***)

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"Laws" - 2

Associative Law of Addition:

a + (b+c) = (a +b) +c

(***NOTE: the order doesn't matter***)

Associative Law of Division:

(a+b)/c = (a /c) + (b/c)

but

a / (b +c) does NOT equal (a/b) + (a/c)

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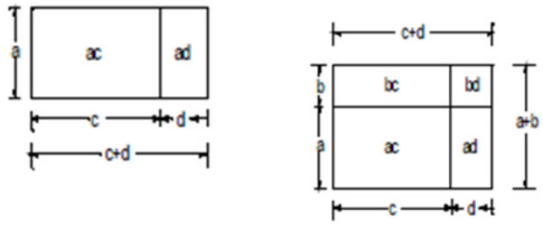
"Laws" - 3

Associative Law of Multiplication:

a(c+d) = ac + ad

Division does not commute: ***a/b does NOT equal b/a unless b = a***



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Positive Exponents

a^2 *means* $a \bullet a = a \times a$

a^3 *means* $a \bullet a \bullet a = a \times a \times a$, *and so forth.*

The ***superscripts*** *2 and 3 are known as* ***exponents****.*

If we multiply a^2 *by* a^3 *we have*

$a^2 \bullet a^3 = (a \bullet a) \times (a \bullet a \bullet a) = a \bullet a \bullet a \bullet a \bullet a = a^5$

when a number raised to an exponent is multiplied by the same number raised to a different exponent, in the product the exponents add.

Note that $a^1 = a$

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Zero Exponents

We have seen that $a^1 = a$

BUT… What about ZERO exponents:

Using the addition of exponents rule:

$$a^n \bullet a^m = a^{n+m}$$

$$a^n \bullet a^0 = a^{n+0} = a^n$$

This equation is the same as

$$a^n \bullet a^0 = a^n \bullet 1$$

Therefore $a^0 = 1$*:*

Any number raised to the zeroth power is equal to one ("unity").

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Negative Exponents

Negative exponents:

using the rule for multiplication of exponents;

$$a^n \bullet a^{-n} = a^{n-n} = a^0 = 1$$

For this equation to be true, we see that

$$a^n \bullet a^{-n} = a^n \bullet 1/a^n = 1$$

therefore…

$$a^{-n} = 1/a^n$$

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Fractional Exponents

Fractional exponents:

The multiplication rule gives us;

$$a^{1/2} \bullet a^{1/2} = a$$

Since $a^{1/2}$ *times itself is* equal to a*,*

we see that $a^{1/2}$ ***must therefore be***

the square root of $a = \sqrt{a}$

Likewise, $a^{1/3}$ is the cube root of a*, etc.*

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Graphing Calculator Alert

Your graphing calculator uses one or more special keys to handle exponents. The **TI-8X** series uses the ^ key to handle raising a variable to an exponent. Your model might have special keys for x^y or y^x. The text typically uses x^y or the ^ key to indicate exponentiation.

Examples: TI: **(1 + 2) [^] 3 = 0.125**

Others: **(1 + 2) [x^y] 3 = 0.125** *or* **(1 + 2) [y^x] 3 = 0.125**

Most calculators also have an x^2 and √ key, the TI units also have a x^{-1} and other special exponent keys (e.g., 10^x, e^x)

Examples:

(- 3) [x^2] = 9

-3 [x^2] = 9

Be sure to check how YOUR calculator does these manipulations…

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Fractional Exponent Problems

Let's try to understand an expression such as $3.3^{1.48}$

which means $(3.3^{1})\ (3.3^{0.4})\ \ (3.3^{0.08})$ =

or $(3.3\ )\ (3.3^{1/10})^{4}\ (3.3^{1/100})^{8}$

Note however, that we do not have to break it down this way to evaluate it since we can enter $3.3^{1.48}$ *directly into a calculator*

To find $3.3^{1.48} = 5.853$:

Press: 3.3 ^ 1.48 <enter>

Result: 5.853297943

Be sure to check how YOUR calculators does these manipulations…

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Questions?

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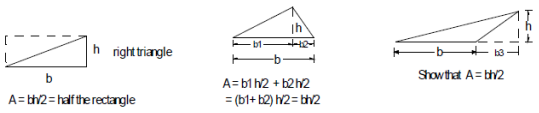
Geometry Review - 1

Area of a triangle: **A = 1/2 base x height**

the triangle's apex doesn't have to lie above the base



A formula can be derived for the **area** of an arbitrary triangle in terms of the lengths of the three sides **x**, **y**, and **z**:

$$A = \frac{1}{4}\sqrt{2x^2y^2 + 2y^2z^2 + 2z^2x^2 - x^4 - y^4 - z^4}$$

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Geometry Review - 2

Circles and Spheres

Circumference of a circle: $C = 2\pi r$

(contains radius length to the 1st power)

Area of a circle: $A = \pi r^2$

(contains radius length to the 2nd power)

Area of a sphere: $A = 4\pi r^2$

(contains radius length to the 2nd power)

Volume of a sphere: $V = (4/3) \bullet \pi r^3$

(contains radius length to the 3rd power)

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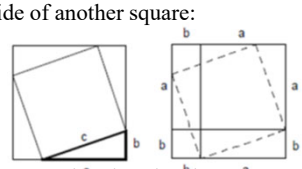
Pythagorean Theorem – Perfect Triangles

Suppose we have a triangle **abc** formed by a **square** with sides **c** inside of another square:



There are four **abc** triangles plus the square on the right making the area of the overall square: $A = 4(\frac{1}{2}\ ab) + c^2$

We evaluate the figure on the left in terms of **a** and **b** only and find its area is comprised of four squares whose total area is: $A = a^2 + b^2 + 2(ab)$

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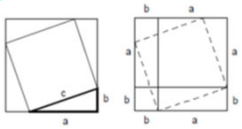
Pythagorean Theorem – Perfect Triangles



Thus we see the large square calculations, produce the equations: $A = 4(\frac{1}{2}\ ab) + c^2 = A = 2ab + c^2$

and $A = a^2 + b^2 + 2ab$

Subtracting 2ab from each side of the equation gives us:

$$a^2 + b^2 = c^2$$

The sum of the squares of the sides equals the square of the hypotenuse…

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Perfect Right Triangles - 2

There is a recipe to find perfect right triangles:

Pick any two integers ***i*** *and* ***j****, where* $i > j$.

The sides of the triangle are i^2+j^2, i^2-j^2, *and* $2ij$

Example: i =2, j=1. Then the hypotenuse = $i^2 + j^2 = 5$,
and the two sides are: $i^2 - j^2 = 3$, and $2ij = 4$

The simplest perfect right triangle has sides 3, 4, and 5.

(Note that $3^2 + 4^2 = 5^2$*)*

Two other perfect right triangles are **5, 12, 13** and **8, 15, 17**.

(*check them*)

Problem to work out on your own (optional):

Use algebra to verify this recipe, that is, show that

$$(i^2-j^2)^2 + (2ij)^2 = (i^2+j^2)^2$$

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Questions?

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Chapter 1.2

The Real Number Line

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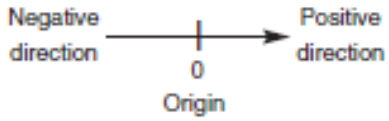
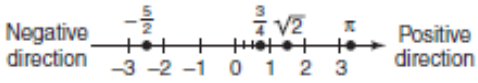
The Real Number Line

Concept



Example



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Inequalities *(Pages 13-14)*

Symbol	Meaning
$<$	Less than
$>$	Greater than
$\leq$	Less than or equal to
$\geq$	Greater than or equal to

Algebraic Expression	Meaning	Equivalent Statement	Geometric Statement
$a > 0$	a is greater than 0.	a is positive.	a lies to the right of the origin.
$a < 0$	a is less than 0.	a is negative.	a lies to the left of the origin.
$a > b$	a is greater than b.	$a - b$ is positive.	a lies to the right of b.
$a < b$	a is less than b.	$a - b$ is negative.	a lies to the left of b.
$a \geq b$	a is greater than or equal to b.	$a - b$ is positive or zero.	a lies to the right of b or coincides with b.
$a \leq b$	a is less than or equal to b.	$a - b$ is negative or zero.	a lies to the left of b or coincides with b.

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Properties of Inequalities *(Page 14)*

Example	Algebraic Expression	Property
Either $2 < 3$, $2 > 3$, or $2 = 3$.	Either $a < b$, $a > b$, or $a = b$.	Trichotomy property
Since $2 < 3$ and $3 < 5$, then $2 < 5$.	If $a < b$ and $b < c$ then $a < c$.	Transitive property
Since $2 < 5$, then $2 + 4 < 5 + 4$ or $6 < 9$.	If $a < b$ then $a + c < b + c$.	The sense of an inequality is preserved if any constant is added to both sides.
Since $2 < 3$ and $4 > 0$, then $2(4) < 3(4)$ or $8 < 12$.	If $a < b$ and $c > 0$, then $ac < bc$.	The sense of an inequality is preserved if it is multiplied by a positive constant.
Since $2 < 3$ and $-4 < 0$, then $2(-4) > 3(-4)$ or $-8 > -12$.	If $a < b$ and $c < 0$, then $ac > bc$.	The sense of an inequality is reversed if it is multiplied by a negative constant.

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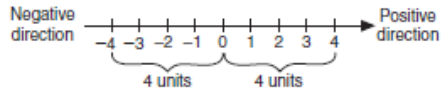
Absolute Values

Distance on a line from -4 to +4:



Definition of the Absolute Value:

$$|a| = \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a < 0 \end{cases}$$

|-4| + |+4| = 4 + 4 = 8

(This is NOT a subtraction!)

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Graphing Calculator Alert

Your calculator may have an absolute value key labeled

ABS

NOTE: the TI series DOES NOT have this key!

If your calculator does have this key be sure to use parentheses to obtain the correct answer…

Examples:

ABS (5 – 2)

ABS (2 – 5)

ABS (3 – 5) – ABS (8 – 6)

ABS (4 – 7) + (-6) …

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Basic Properties of Absolute Value

Example	Algebraic Expression	Property
$\|-2\| \geq 0$	$\|a\| \geq 0$	Absolute value is always nonnegative.
$\|3\| = \|-3\| = 3$	$\|a\| = \|-a\|$	The absolute values of a number and its negative are the same.
$\|2-5\| = \|-3\| = 3$ $\|5-2\| = \|3\| = 3$	$\|a-b\| = \|b-a\|$	The absolute value of the difference of two numbers is always the same, irrespective of the order of subtraction.
$\|(-2)(3)\| = \|-2\|\|3\| = 6$	$\|ab\| = \|a\|\|b\|$	The absolute value of a product is the product of the absolute values.

Table 8 (Page 17)

 45

Chapter 1.3
Algebraic Expressions and Polynomials

 46

Algebraic Expressions

"You invest p dollars at 6% simple annual interest for 1 year.

How much do you now have?"

- Variables (p)
- Constants (0.06)
- Algebraic Operations (+, - , /, x)
- Resultant (?)

Resulting in…

p + 0.06p = ? or **p(1 + 0.06) = ?**

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Polynomials

Polynomials are equations of the form:

$$\mathbf{P = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \ldots a_1 x^1 + a_0}$$

where:

Coefficients $\mathbf{a_x}$ are constant real numbers

$\mathbf{a_n \neq 0}$

n = nonnegative integer

and the $\mathbf{P_{n-a}}$ term = $\mathbf{(a_{n-a} x^{n-a})}$

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The Degree of a Polynomial

The degree of a polynomial is the exponent value of the highest monomial with a nonzero coefficient:

$x + y$ ← Degree 1

xy ← Degree 2

$2x^2y + y^2 - 3xy + 1$ ← Degree 3

$3x^4 + xy - y^3$ ← Degree 4

Polynomials are equal if all their terms are equal

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Questions?

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Chapter 1.4

Factoring

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Common Factors

$x^2 + x = x(x + 1)$

$2xy + 2 = 2(xy + 1)$

$25x^3 - 10x^2 = 5x^2(5x - 2)$

$3x^4 + x^3 + xy = x(3x^3 + x^2 + y)$

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Factoring by Grouping

GIVEN: $2ab + b + 2ac + c = ?$

1. **Grouping b, c:**

$(2ab + b) + (2ac + c)$

2. **Common factors b and c:**

$b(2a+1) + c(2a+1)$

3. **Common factors (2a+1):**

$(b+c)(2a+1) =$ ***final answer***

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Factoring 2nd Degree Polynomials

GIVEN:

Factor the expression $x^2 - 7x + 10$

NOTE: The constant is positive (10) and the middle term(7) is negative therefore we expect the two roots of this equation to be negative

Integer Pairs products of 10: 1 & 10 *or* 2 & 5

Factoring we get:

$x^2 - 7x + 10 = (x-2)(x-5)$

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Factoring 2nd Degree Polynomials

GIVEN:

Factor the expression $x^2 - 9$

Note: This is a difference of two squares; x^2 and 9 ($= 3^2$) !

Factoring we get:

$$x^2 - 9 = (x+3)(x-3)$$

In general for $a^2-b^2 = (a+b)(a-b)$

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Sum/Difference of Cubes

GIVEN: Sum of cubes:

$$a^3+b^3$$
$$= (a+b)(a^2-ab+b^2)$$

GIVEN: Difference of cubes:

$$a^3-b^3$$
$$= (a-b)(a^2+ab+b^2)$$

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"MAGICAL" Factoring for Second-Degree Polynomials

This is NOT a required Technique…

Factoring involves a certain amount of trial and error that can become frustrating, especially when the lead coefficient is not 1. We demonstrate the method for the polynomial $4x^2+11x+6 = ?$ ***Eq. 1***

(1) Using the lead coefficient of 4, write the pair of incomplete factors

$(4x \pm ?)\ (4x \pm ?)$ ***Eq. 2***

(2) Next, multiply the coefficient of x^2 and the constant term in Equation (1) to produce 4 • 6 = 24. Now find two integers whose product is 24 and whose sum is 11, the coefficient of the middle term of (1). Since 8 and 3 work, and all signs are +, we can write

$(4x + 8)(4x + 3)$ ***Eq. 3***

Finally, within each parenthesis discard any common numerical factor. (Discarding a factor may only be performed in this "magical" type of factoring.) Thus (4x + 8) reduces to (x + 2) and we write

$(x + 2)(4x + 3)$ ***Eq. 4***

which is the factorization of 4x2 + 11x + 6

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Irreducible Polynomials

Prime or irreducible polynomials cannot be written as a product of two polynomials of positive degree…

Examples:

$$x^2+1$$
$$x^2+x+1$$

knowing these makes it easier to know when to 'stop' factoring…

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Questions?

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Chapter 1.5

Rational Expressions

 60

Rational Expressions

The reciprocal of (b/a) equals (a/b)

Multiplication of rational expressions

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$$

The Rule of 'One': a/a = 1; [d/c / d/c] =1

$$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{\frac{a}{b}}{\frac{c}{d}} \cdot \frac{\frac{d}{c}}{\frac{d}{c}} = \frac{\frac{a}{b}\frac{d}{c}}{\frac{c}{d}\frac{d}{c}} = \frac{a}{b}\frac{d}{c}$$

 61

Reduction by Cancellation

$$\frac{4xy}{4yz} =$$

1. Cancel the 4: $\frac{\cancel{4}xy}{\cancel{4}yz} = \frac{xy}{yz}$

2. Cancel the y: $\frac{x\cancel{y}}{\cancel{y}z} = \frac{x}{z}$

 62

Reduction by Cancellation

$$\frac{(2x^2+3)(4x^3+4)}{(7x^7+1)(2x^2+3)} =$$

Cancel the $(2x^2+3)$ terms:

$$\frac{\cancel{(2x^2+3)}(4x^3+4)}{(7x^7+1)\cancel{(2x^2+3)}} = \frac{(4x^3+4)}{(7x^7+1)}$$

 63

Add/Subtract Fractions

Multiplying by '1' (7/7, 3/3)

$$\frac{2}{3}+\frac{1}{7} = \frac{2}{3}\frac{7}{7}+\frac{1}{7}\frac{3}{3} = \frac{14}{21}+\frac{3}{21} = \frac{17}{21}$$

Multiplying by '1' (**7/7, (b+6)/(b+6)**)

$$\frac{2a+5}{b+6}+\frac{a-4}{7} = \frac{2a+5}{b+6}\frac{7}{7}+\frac{a-4}{7}\frac{b+6}{b+6} =$$

$$\frac{7(2a+5) + (a-4)(b+6)}{7(b+6)}$$

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Cross Multiplication

Multiply the left-hand numerator by the right-hand denominator and visa versa

GIVEN: $\frac{x}{y} = \frac{a}{b}$

This is the same as: $xb = ya$

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Chapter 1.6

Integer Exponents

review on your own…

 66

Chapter 1.7

Rational Exponents and Radicals

 67

Properties of Powers & Roots

For $a^n = b$ and $a = b^{1/n}$ for $n>0$

Example		*Property*
$2^3 = 8$	$(-2)^3 = -8$	Any power of a real number is a real number.
$8^{1/3} = 2$	$(-8)^{1/3} = -2$	The odd root of a real number is a real number.
$0^n = 0$	$0^{1/n} = 0$	A positive power or root of zero is zero.
$4^2 = 16$	$(-4)^2 = 16$	A positive number raised to an even power equals the negative of that number raised to the same even power.
$(16)^{1/2} = 4$		The principal root of a positive number is a positive number.
$(-4)^{1/2}$ is undefined in the real number system.		The even root of a negative number is not a real number.

This is Table 11, Page 58

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Properties of Radicals

Example	*Property*
$\sqrt[3]{8^2} = (\sqrt[3]{8})^2 = 4$	$\sqrt[n]{b^m} = (\sqrt[n]{b})^m$
$\sqrt{4}\sqrt{9} = \sqrt{36} = 6$	$\sqrt[n]{a}\sqrt[n]{b} = \sqrt[n]{ab}$
$\frac{\sqrt[3]{8}}{\sqrt[3]{27}} = \sqrt[3]{\frac{8}{27}} = \frac{2}{3}$	$\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$
$\sqrt[3]{(-2)^3} = -2$	$\sqrt[n]{a^n} = a$ if n is odd
$\sqrt{(-2)^2} = \lvert -2 \rvert = 2$	$\sqrt[n]{a^n} = \lvert a \rvert$ if n is even

This is Table 12, Page 62

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More Difficult Problems

If you take out a mortgage for $49,000 for 10 years at an interest rate of 5.25%, how much is the monthly payment?

where ***m is the monthly payment,***
P is the original principal (49,000),
I is the interest rate (0.0525),
and ***Y is the number of years (10)***

$$m = \frac{P\frac{I}{12}}{1-\left(1+\frac{I}{12}\right)^{-12\,Y}} +$$

This problem is much more difficult, but still within the scope of this course

FYI: The answer is m = $625.73

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Quick Problems - 1

Ch. 1.1, Pg 12 #63.

A woman's take-home pay is $210.00 after deducting 18% withholding tax. What is her pay before the deduction?

You have 5 minutes to solve this

Try to do this before you look at the answer on the next slide…

 71

Quick Problems – 1 ANS

Ch. 1.1, Pg 12 #63.

A woman's take-home pay is $210.00 after deducting 18% withholding tax. What is her pay before the deduction?

ANSWER:

18% is deducted from 100% pay leaving 100% – 18% = 82% for net pay. Thus, 210 is 82% of her gross earnings:

210 = 0.82x

x = 210/0.82 = **$256.10** *(note the '$' in the answer…)*

*This is the question type where **Net = (%) Gross**.*

*To find the amount of tax: **Gross – Net = Tax***

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Quick Problems - 2

Ch. 1.1, Pg 12 #66.

Eric starts at a certain time driving his car from New York to Philadelphia going 50 mph. Sixty minutes later, Steve leaves in his car en route from Philadelphia to New York going 40 mph.

When the two cars meet, which one is nearer to New York?

You have 5 minutes to solve this

Try to do this before you look at the answer on the next slide…

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Quick Problems - 2

Ch. 1.1, Pg 12 #66.

Eric starts at a certain time driving his car from New York to Philadelphia going 50 mph. Sixty minutes later, Steve leaves in his car en route from Philadelphia to New York going 40 mph.

When the two cars meet, which one is nearer to New York?

ANSWER:

The distance from Albany, NY to Philly is about 230 miles.

Although we could put a timeline to try to show where the vehicles are at time say ***t = 60*** *minutes,* ***NOTE: when the cars meet they are BOTH the same distance from both NY and Philly.***

This was a trick question…

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Quick Problems - 3

Ch. 1.3, Pg 29 #60.

Perform the indicated operation:

$$5(2x - 3)^2$$

You have 5 minutes to solve this

Try to do this before you look at the answer on the next slide…

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Quick Problems - 3

Ch. 1.3, Pg 29 #60.

Perform the indicated operation:

$$5(2x - 3)^2$$

ANSWER:

$$= 5\,(2x - 3)\,(2x - 3)$$
$$= 5(4x^2 - 12x + 9)$$
$$= (20x^2 - 60x + 45) \text{ final answer}$$

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Quick Problems - 4

Ch. 1.3, Pg 29 #62.

Perform the indicated operation:

$$(x - 1)\,(x + 2)\,(x + 3)$$

You have 5 minutes to solve this

Try to do this before you look at the answer on the next slide…

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Quick Problems - 4

Ch. 1.3, Pg 29 (5^{th} 28) #62. Perform the indicated operation:

$$(x - 1)\,(x + 2)\,(x + 3)$$

ANSWER:

$$= (x^2 + x - 2)\,(x + 3)$$
$$= x^2\,(x + 3) + x\,(x + 3) - 2\,(x + 3)$$
$$= x^3 + 3x^2 + x^2 + 3x - 2x - 6$$
$$= x^3 + 4x^2 + x - 6 \quad \text{final answer}$$

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Quick Problems - 5

Ch. 1.3, Pg 29 #63

An investor buys *x shares of IBM stock at* $98 per share at Thursday's opening of the stock market. Later in the day, the investor sells *y shares of AT&T stock at $38 per* share and *z shares of TRW stock at $20 per* share. Write a polynomial that expresses the amount of money the buyer has invested at the end of the day.

You have 5 minutes to solve this

Try to do this before you look at the answer on the next slide…

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Quick Problems - 5

Ch. 1.3, Pg 29 #63

An investor buys *x shares of IBM stock at* $98 per share at Thursday's opening of the stock market. Later in the day, the investor sells *y shares of AT&T stock at $38 per* share and *z shares of TRW stock at $20 per* share. Write a polynomial that expresses the amount of money *the buyer has invested at the end of the day.*

ANSWER:

he pays for IBM stock: **– 98*x***

he sells AT&T stock: **+ 38*y***

he sells TRW stock: **+ 20*z***

INVESTMENT = - 98x + 38y +20z final answer

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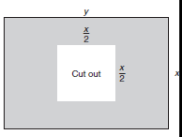
Quick Problems - 6

Ch. 1.3, Pg 30 #64.

An artist takes a rectangular piece of cardboard whose sides are x and y and cuts out a square of side x/2 to obtain a mat for a painting, as shown in Figure 5. Write a polynomial giving the area of the mat.

You have 5 minutes to solve this



Try to do this before you look at the answer on the next slide…

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Quick Problems - 6

Ch. 1.3, Pg 30 #64.

An artist takes a rectangular piece of cardboard whose sides are x and y and cuts out a square of side x/2 to obtain a mat for a painting, as shown in Figure 5. Write a polynomial giving the area of the mat.

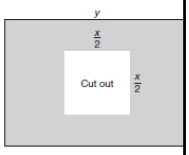
ANSWER:

$\text{Area}_{mat} = \text{Area}_{outer} - \text{Area}_{inner}$

$= xy - (x/2)(x/2)$

$= xy - (x/2)^2$

$= xy - (x^2/4)$ ***final answer***



 82

Quick Problems - 7

Ch. 1.3, Pg 30 #73.

Perform the multiplication mentally:

$(3x - 1)^2$

You have 3 minutes to solve this

Try to do this before you look at the answer on the next slide…

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Quick Problems - 7

Ch. 1.3, Pg 30 #73.

Perform the multiplication mentally:

$(3x - 1)^2$

ANSWER:

$= (3x - 1)(3x - 1)$

$= 9x^2 - 3x + 1 - 3x$

$= 9x^2 - 6x + 1$ ***final answer***

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Quick Problems - 8

Ch. 1.3, Pg 30 #74.

Perform the multiplication mentally:

$(x + 2)(x - 2)$

You have 3 minutes to solve this

Try to do this before you look at the answer on the next slide…

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Quick Problems - 8

Ch. 1.3, Pg 30 (5th 28) #74.

Perform the multiplication mentally:

$(x + 2)(x - 2)$

ANSWER:

$= x(x - 2) + 2(x - 2)$

$= x^2 - 2x + 2x - 4$

$= x^2 - 4$

(This is a difference of squares) final ans.

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Homework Set #02

Section 1.3 (Expressions and Polynomials)

Page 28: Problems 4, 27, 42, 44, **49**, 50, **58**, **82**

Section 1.4 (Factoring)

Page 38: Problems 2, **9**, 11, 12, 13, 32, 40

Section 1.5 (Rational Expressions)

Page 47: Problems 1, 2, 7, 8, 25, 32, 51, 52

Section 1.7 (Rational Exponents and Radicals)

Page 67: Problems 1, 5, 6, 19, 20

Red problems used for homework quiz

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Topics in Session 3

Ch. 1 The Foundations of Algebra

1.8 Complex Numbers

(NOTE: There are normally NO complex numbers in CM/FM problems. Complex numbers DO exist in electrical and mechanical engineering…)

Chapter 1 Review

Ch. 2 Equations and Inequalities

2.1 Linear Equations in One Unknown

2.2 Applications: From Words to Algebra

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Due Next Class:

Next Monday 7 September is LABOR DAY
NO CLASSES

***Due* – Session 3 (*14 September*):**

- ***DUE:* Textbook readings**
- ***Lecture and Problem Review***
- ***Homework Set #02:* Due by 12:00Noon**
- ***'Quiz' on Homework Set #02***

***Due* – Session 4 (*21 September*):**

- ***Nomenclature, Notes***
- ***Lecture and Problem Review***
- ***Homework Set #03:* Due by 12:00Noon**
- ***'Quiz' on Homework Set #03***

NOTE: NO CLASS on Monday 7 September

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Any Questions?
Send me an email …

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or
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End

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