Pratt

Math 150 – Fall 2020
Algebra & Trigonometry

Charles Rubenstein, Ph. D.
Professor of Engineering & Information Science

Session 3: Monday 9/14/20
6:30pm - 9:20pm
via REMOTE LEARNING
Revision 1

Instructor Contact Information

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Fall 2020 Virtual Office hours *ONLY*
Wednesdays 10:00am-2:00pm via Zoom Meeting
To make your appointment
Send me an email at least one day in advance :
c.rubenstein@ieee.org
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This Class Session

Class Sessions Posted Online Friday before Class
- ***DUE:*** ***Homework Set #02 by 12:00Noon 14 September!***
- ***NOTE: Quiz 2 = four problems from hwk***
- ***Due:*** ***Textbook readings***
- ***Lecture: Solving Linear and Quadratic Equations of One Variable***
- ***Review:*** ***Homework Set #02***

***In class* – Session 4: Monday 21 September:**
- ***DUE:*** ***Homework Set #03 by 12:00Noon 21 September!***
- ***NOTE: Quiz 3 = four problems from hwk***
- ***Review:*** ***Textbook readings***
- ***Lecture: Solving Equations of Two Variables***

NO CLASSES Monday - 28 September = Instructor's Holiday
Exam #1 Monday – 5 October via email

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About Emailing me your Homework

Homework is due not later than Noon on class days.
1. Use ***DARK, BLACK pencil or pen.***
   If I can't read your work you get a ZERO!
2. Please ***scan your work*** as a PDF and save it as ***lastname_xx.pdf***
   HOWEVER – IF YOU CAN NOT SCAN –
   a. Take photo(s) of your work and insert the photo(s) into a Word (***rtf***, ***doc***, ***docx***) document
   b. Save as ***lastname_xx.docx*** (etc.), where ***xx*** is your assignment number.

Then email your file to me:
c.rubenstein@ieee.org
With the subject line ***Math Homework #XX.***
Email me ONLY the requested four (4) problems.
(Email any you might be challenged by in a separate document)

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About the Homework Quizzes

Since there are dozens of practice problems within each homework set, I feel it unnecessary to have you send ALL the problems to me each week.
I have selected four (4) problems from each homework for you to submit and - as long as at least three are answered correctly - receive 'quiz' credit of 3% for correct answers.
These are the selected problems for the next homework assignments:

HWK #03: Ch. 1 Review Section: 17; Section 2.2: 1, 4, 12
HWK #04: Section 2.2: 25, 28, 48 and Section 2.3: 16
HWK #05: Section 2.4: 1, 4, 8, 21
HWK #06: Section 2.2: 54a, 54c, 55, 57

Today's Homework was due not later than Noon today.
The other homeworks are due each of the following weeks at Noon.

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About Exam #1 – Worth 20%

Exam #1 is a one hour exam

It will be emailed to you on Monday 5 October by 12:00Noon

EXAM 1 is DUE
by 4:00pm on 5 October!

See HOW TO EMAIL ME YOUR EXAM on the next slide

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About Emailing me your Exam(s)

Exams will be emailed to you not later than 12:00pm NOON (ET) and is due not later than 4:00pm that day.

(Exam 1: 5 October; Exam 2: 9 November; Final 7 December)

1. *Use **DARK, BLACK pencil or pen**.*
   If I can't read your work you get a ZERO!
2. *Please **scan your work** as a PDF and save it as **lastname_EE.pdf***
   HOWEVER – IF YOU CAN NOT SCAN –
   a. *Take photo(s) of your work and insert the photo(s) into a Word (**rtf**, **doc**, **docx**) document*
   b. *Save as **lastname_EE.docx** (etc.), where **EE** is your exam 'ID' – **E1**, **E2** or **FE**(final exam).*

Then email your file** to me: **c.rubenstein@ieee.org

*With the subject line **Exam EE**.*

Email me BOTH the worked out problems AND solutions.
Be sure to include any units…

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Math 150 – Class Topics

1. **The Foundations of Algebra**
2. **Equations and Inequalities**
3. **Functions**
4. **Polynomial Functions**
5. **Rational Functions and Conic Sections**
6. **Exponential and Logarithmic Functions**
7. **The Trigonometric Functions**
8. **Analytic Trigonometry**
9. **Applications of Trigonometry**
10. **Systems of Equations and Inequalities**
11. **Matrices, Linear Systems, and Determinants**
12. **Topics in Algebra**

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Draft Schedule: Math 150 – Fall 2020 – Remote Learning

Monday	Notes
24-Aug	1. Introduction: Numbers, Arithmetic Operations, Fractions
31-Aug	2. Manipulation of Algebraic Expressions; *Hwk #1 Due @ Noon*
7-Sep	NO CLASSES – Labor Day
14-Sep	3. Solving Linear and Quadratic Equations of One Variable; *Hwk #2 Due*
21-Sep	4. Solving Equations of Two Variables; *Hwk #3 Due*
28-Sep	NO CLASSES – Instructor Holiday
5-Oct	5. Creating Equations – Polynomial Functions; *Hwk #4 Due; Exam #1*
12-Oct	6. Polynomial Functions, continued; *Hwk #5 Due; Exam 1 Review*
19-Oct	7. Functions, Graphing, Exponents and Logarithms; *Hwk #6 Due*
26-Oct	8. Trigonometric Functions, Pythagorean Theorem; *Hwk #7 Due*
2-Nov	9. Applications of Trigonometry; *Hwk #8 Due*
9-Nov	10. Analytic Trigonometry: Identities & Graphing; *Hwk #9 Due; Exam #2*
16-Nov	11. Areas and Volumes of Geometric Solids; *Hwk #10 Due; Exam 2 Review*
23-Nov	12. Systems of Equations and Inequalities;
30-Nov	13. Series and Sequences, Review topics
7-Dec	Final Examination *Exam Emailed by 12:00Noon - Due at 5:00pm ET*

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www.CharlesRubenstein.com/150

20fa03.pdf = *This slide set**

20fa03_h.pdf = *slides as 6-up handouts**

**My goal is to post these not later than Noon on the Friday one week before our Zoom Class Meetings*

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Homework #01
Selected
Review Problems

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Problems – Homework #1

Ch. 1.1, Pg 10-11

In Exercises 37–40, find a counterexample; that is, find real values for which the statement is false.

38 (***Also find values for a and b for which the statement is true***.)

a/b = b/a

FALSE: let a=1, b=2: ½ = 2 = *Not True!*

FALSE: let a=2, b=4: 2/4 = 4/2 = *Not True!*

TRUE: whenever a = b

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Problems – Homework #1

Ch. 1.1, Pg 10-11

In Exercises 37–40, find a counterexample; that is, find real values for which the statement is false.

40 (a + b) (c + d) = ac + bd

FALSE:

let a=2, b=3, c=4, d=5:

then **(a + b) (c + d) = (2+3)(4+5) = 5*9 = 45**

and **ac + bd = 2*4+3*5 = 8+15 = 23**

***However:** 45 does NOT equal 23 !*

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Problems – Homework #1

Ch. 1.1, Pg 10-11

57 On a map of Pennsylvania, 1 inch represents 10 miles.

Find the distance represented by 3.5 inches.

Inches/Miles: 1/10 = 3.5/x

Cross multiply: 1x = 10 • 3.5

x = 35 miles

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Problems – Homework #1

Ch. 1.1, Pg 10-11

58. A car travels 135 miles on 6 gallons of gasoline.

How far can it travel on 10 gallons of gasoline?

$$\frac{\text{miles}}{\text{gallons}} = \frac{135}{6} = \frac{x}{10}$$

Cross multiplying:

135 • 10 = 6 • x

6x = 1350 ; x = 1350/6

x = 225 miles

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Problems – Homework #1

Ch. 1.1, Pg 10-11

#61. Which is the better value: 1 pound 3 ounces of beans for 85 cents, or, 13 ounces for 56 cents?

Convert to ounces:

1 pound, 3 ounces = 16 + 3 ounces = 19 ounces

price per unit of measure:

Compare $\frac{price}{ounce}$: $\frac{85}{19}$ = 4.47 ¢/oz.

Compare $\frac{price}{ounce}$: $\frac{56}{13}$ = 4.31 ¢/oz.

13 ounces for 56 cents = best buy

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Problems – Homework #1

Ch. 1.1, Pg 10-11

#62. A piece of property is valued at $28,500.

What is the real estate tax at $75.30 per $1000.00 evaluation? $\frac{tax}{value} : \frac{75.30}{1000} = \frac{x}{28500}$

Cross multiply (75.30)(28500) = 1000 x

2,146,050 = 1,000 x

2,146.05 = x

The real estate tax is **$2,146.05**

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Homework #02
Selected
Review Problems

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Problems – Homework #2

Ch. 1.4, Pg 38 (5th 36)

#2. **Factor completely… 1 / 4 x + 3 / 4 y**

1 / 4 x + 3 / 4 y =

1 / 4 (x + 3y) ***final ans.***

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Problems – Homework #2

Ch. 1.5, Pg 47 (5th 44) #1, 2, 7, 8, 32, 52

Prob. 1. $\frac{x+4}{x^2-16}$

Since (x^2 – 16) = (x+4) (x-4) = *difference of squares we can write*

$$\frac{x+4}{x^2-16} = \frac{\cancel{(x+4)}}{\cancel{(x+4)}\,(x-4)} = \frac{1}{(x-4)}$$

final ans

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Problems – Homework #2

Ch. 1.5, Pg 47 (5th 44) #1, 2, 7, 8, 32, 52

Prob. 2. $\frac{(y^2-25)}{(y+5)}$

Since (y^2 – 25) = (y+5) (y-5) we can write

$$\frac{\cancel{(y+5)}\,(y-5)}{\cancel{(y+5)}}$$

which simplifies to

= (y-5) ***final ans.***

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Problems – Homework #2

Ch. 1.5, Pg 47 (5th 44) #1, 2, 7, 8, 32, 52

Prob. 7. $\frac{2}{3x-6} \div \frac{3}{2x-4}$

$$= \frac{\frac{2}{3x-6}}{\frac{3}{2x-4}} = \frac{\frac{2}{3(x-2)}}{\frac{3}{2(x-2)}} \cdot \frac{(6)(x-2)}{(6)(x-2)}$$

$$= \frac{4}{9},\ x \neq 2$$ ***final ans.***

(Note: x≠2 since (x-2) would then be zero, undefined!)

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Problems – Homework #2

Ch. 1.5, Pg 47 (5th 44) #1, 2, 7, 8, 32, 52

Prob. 8.

$$\frac{5x+15}{8} \div \frac{3x+9}{4} = \frac{\frac{5(x+3)}{8}}{\frac{3(x+3)}{4}}$$

$$= \frac{\frac{5(x+3)}{8}}{\frac{3(x+3)}{4}} \cdot \frac{(8)}{(8)} = \frac{5(x+3)}{6(x+3)}$$

$$= \frac{5}{6},\ x \neq -3$$ ***final ans.***

(Note: x≠3 since (x-3) would then be zero, undefined!)

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Problems – Homework #2

Ch. 1.5, Pg 47 (5th 44) **Prob. 32.**

$$\frac{x}{x^2-4}+\frac{2}{4-x^2} = \frac{x}{x^2-4}+\frac{2}{(-1)(x^2-4)}$$

$$= \frac{x}{x^2-4}-\frac{2}{x^2-4}$$

$$= \frac{x-2}{x^2-4}$$

$$= \frac{x-2}{(x+2)(x-2)}$$

$$= \frac{1}{x+2}$$ ***final ans.***

Problems – Homework #2

Ch. 1.5, Pg 47 (5th 44) #1, 2, 7, 8, 32, 52

Prob. 52. $\dfrac{x-\frac{1}{x}}{2+\frac{1}{x}}$

$$= \frac{x-\frac{1}{x}}{2+\frac{1}{x}} \cdot \frac{(x)}{(x)}$$

$$= \frac{x^2-1}{2x+1}$$ ***final ans.***

Problems – Homework #2

Ch. 1.7, Pg 67 (5th 63) # 5, 6

Prob. 5. $\dfrac{2x^{1/3}}{x^{-3/4}}$ *we note that* $\mathbf{1/x = x^{-1}}$

Therefore the equation is: $(\mathbf{2x^{1/3}})(x^{(-(\frac{3}{4}))})$

We also know: $\mathbf{(x^a)(x^b) = x^{(a+b)}}$

Therefore we now have: $(\mathbf{2x^{(\frac{1}{3}-(-(\frac{3}{4})))}})$

Find the common denominator (12) we get

$$\mathbf{2x^{(\frac{4}{12}+\frac{9}{12})}}$$

and… $\mathbf{2x^{(\frac{13}{12})}}$ ***final ans.***

Problems – Homework #2

Ch. 1.7, Pg 67 (5th 63) # 5, 6

Prob. 6. $\dfrac{y^{-2/3}}{y^{1/5}}$ *noting that* $\mathbf{1/y = y^{-1}}$

Therefore the equation is: $(\mathbf{y^{-2/3}})(y^{-(\frac{1}{5})})$

We also know: $\mathbf{(y^a)(y^b) = y^{(a+b)}}$

Therefore we now have: $(\mathbf{y^{(\frac{-2}{3}+(-\frac{1}{5}))}})$

Find the common denominator (15) we get

$$\mathbf{y^{(\frac{-10}{15}-\frac{3}{15})}}$$

And thus… $\mathbf{y^{(-\frac{13}{15})} = 1 / y^{\frac{13}{15}}}$ ***final ans.***

Questions?

Topics in Session 3

Ch. 1 The Foundations of Algebra

1.8 Complex Numbers

(NOTE: There are normally NO complex numbers in CM/FM problems. Complex numbers DO exist in electrical and mechanical engineering…)

Chapter 1 Review

Ch. 2 Equations and Inequalities

2.1 Linear Equations in One Unknown

2.2 Applications: From Words to Algebra

In Chapter 2

Equations and Inequalities
2.1 Linear Equations in One Unknown
2.2 Applications: From Words to Algebra
2.3 The Quadratic Equation
2.4 Applications of Quadratic Equations
2.5 Linear and Quadratic Inequalities
2.6 Absolute Value in Equations and Inequalities

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Chapter 2

2.2 Applications: From Words to Algebra

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From Words to Algebra

1. Read the problem through the first time to get a general idea of what is being asked.
2. Read the problem a second time to recognize what may be important in determining that which is to be found.
3. If possible, estimate the solution to this problem, and then compare this estimate with your final answer.
4. Let some algebraic symbol denote the quantity to be found.
5. If possible, represent other quantities in the problem in terms of the algebraic symbol designated in Step 4.
6. Find various relationships (equations or inequalities) in the problem.
7. Use relationships established in Step 6 to find the solution to the problem.
8. Verify that your answer is, indeed, the solution to the problem.

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Key Word Translations

Word or Phrase	Algebraic Symbol	Example	Algebraic Expression
Sum	+	Sum of two numbers	$a + b$
Difference	−	Difference of two numbers	$a - b$
		Difference of a number and 3	$x - 3$
Product	× or ·	Product of two numbers	$a \cdot b$, $(a)(b)$, or ab
Quotient	÷ or /	Quotient of two numbers	$\frac{a}{b}$, a/b, or $a \div b$
Exceeds		a exceeds b by 3	$a = b + 3$
More than		a is 3 more than b	or
More of		There are 3 more of a than of b.	$a - 3 = b$
Twice		Twice a number	$2x$
		Twice the difference of x and 3	$2(x - 3)$
		3 more than twice a number	$2x + 3$
		3 less than twice a number	$2x - 3$
Is or equals	=	The sum of a number and 3 is 15.	$x + 3 = 15$

Table 1, Page 96

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Typical Word Problem Types

- Prices Problems and Discounts Problems
- Coin Problems
- Interest Problems
- Distance Problems
- Travel Problems
- Mixture Problems
- Work Problems
- Formulas

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Prices and Discounts Problems

"If you pay for something after receiving a discount, what was the price before the discount?"

SOLUTION:

Let **p** = PRICE before the discount.

Let **d** = DISCOUNT as a fraction

Then: **(d)p** = Amount discounted and

C = COST (the price AFTER the discount)

C = p – (d)p

If you pay $6000 for a car after receiving a 25% discount, what was the price of the car before the discount?

$6000 = p – (0.25)p 0.75p = $6000 p = $6000/.75

p = $8000 ***final answer MUST include units…***

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Coin Problems

Always distinguish between the number of coins and the value of the coins. You may also find it helpful to use a chart.

"You have AMOUNT in VALUE of coins. If there are n more of one than the other, how many coins of each type are there?"

SOLUTION:

Let **A**= Amount in cents , **q** represents the number of VALUE, V_1 ,

Then **q** = Quantity V_1 and **(q – n)** = Quantity V_2 and

$$A = V_1 q + V_2 (q - n)$$

$$A = q (V_1 + V_2) - n V_2 \quad \text{and} \quad A + n V_2 = q (V_1 + V_2)$$

$$q = \frac{(A + n V_2)}{(V_1 + V_2)} \qquad \textit{Then solve for (q-n)}$$

Coin	Number	Value	Total Value
Coin #1	q	V_1	V_1 q
Coin #2	(q – n)	V_2	V_2 (q – n)

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Coin Problem Example

"You have $32.00 in quarters and dimes. If there are 30 more quarters than dimes, how many coins of each type do you have?

SOLUTION:

Let **A**= Amount in cents, **q** represents the number of one VALUE, V_1 ,

Then **q = Quantity V_1** and **(q – n) = Quantity V_2** and

$$3200 = 25\,q + 10\,(q - 30)$$

$$3200 = q\,(25 + 10) - 300 \quad \text{and} \quad 3200 + 300 = q\,(35)$$

$$q = \frac{(3500)}{(35)} \qquad \mathbf{q = 100 \text{ Quarters, and thus } 70 \text{ dimes}}$$

Coin	Number	Value	Total Value
Quarters	q	$V_1 = 25$	25 q
Dimes	(q – n)	$V_2 = 10$	10 (q – 30)

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Interest Problems

Simple Interest: Interest is the fee charged for borrowing money.

Simple interest assumes the fee to be a fixed percentage ***r*** of the principal ***P*** (the amount borrowed) for one time period, ***t***.

The interest due at the end of each year is **Pr**, and the total interest ***I*** due at the end of **t** years is

$$I = Prt$$

If ***S*** is the total amount owed at the end of ***t*** years, then

S = P + I such that: $S = P + Prt$

since both the principal and interest need to be repaid.

Often problems include multiple loans.

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Interest Problem Example

"A part of $7000 was borrowed at 6% simple annual interest and the remainder at 8%. If the total amount of interest due after 3 years is $1380, how much was borrowed at each rate?"

SOLUTION: using *I = Prt*

Let **s** = amount borrowed at **6%**

Then **7000 - s** = the amount borrowed at **8%**

Loan	P	r	t	I
6% portion	*s*	*0.06*	*3*	*0.18s*
8% portion	*7000 - s*	*0.08*	*3*	*0.24(7000-s)*

$$\$1380 = 0.18s + 0.24(7000 - s)$$

$$\$1380 = 0.18s + 1680 - 0.24s$$

$$0.06s = \$300 \quad \text{and } s = \$5000 \text{ at } 6\%$$

and therefore $7000-5000 = **$2000 at 8%**

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Distance & Travel Problems

Distance problems take the form of

Distance = (Rate) (Time)

$$d = r \bullet t$$

Setting up distance problems requires you to evaluate:

1. Are the distances equal?
2. Are the sum/difference of the distances constant?

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Distance & Travel Example

"Two trains leave New York for Chicago – an 800 mile trip.

*The **first train** travels at an average speed of **60 mph**. The **second train**, which **departs an hour later**, travels at an average speed of **80 mph**. **How long will it take the second train to overtake the first train?**"*

***t** = the number of hours the second train travels*

***t + 1** = the number of hours the first train travels*

Train	Rate	Time	Distance
t_1	60	*t + 1*	*60(t + 1)*
t_2	80	*t*	*80t*

They travel same amount when they meet:

$$60(t + 1) = 80t \quad \textit{thus} \quad 60t + 60 = 80t$$

$$60 = 20t \quad \textit{and} \quad 3 = t$$

and it takes the second train 3 hours to catch up… final ans

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Mixture Problems

Mixture problem involve mixing varieties of a commodity, say two or more types of metal, to obtain a mixture with a desired value.

If the commodity is measured in pounds, the relationships we need are as follows:

(Number of pounds)(Price per pound) = Value of commodity

Sum of weights of all varieties = Weight of mixture

Sum of values of all varieties = Value of mixture

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Mixture Problem Example

"How many pounds of Brazilian coffee worth $10 per pound must be mixed with 20 pounds of Colombian coffee worth $8 per pound to produce a mixture worth $8.40 per pound?"

SOLUTION: Let **B** = number of pounds of Brazilian coffee.

We display all the information, using cents in place of dollars.

Type of Coffee	Number of pounds	Price/pound	Value (cents)
Brazilian	***B***	***1000***	***1000B***
Columbian	***20***	***800***	***16,000***
Mixture	***B + 20***	***840***	***840(B + 20)***

value of mixture = (value of Brazilian) + (value of Colombian)

$$840(B + 20) = 1000B + 16{,}000$$

$$840B + 16{,}800 = 1000B + 16{,}000$$

$$800 = 160B$$

thus **5 = B** and we must **add 5 pounds of Brazilian coffee**

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Work Problems

Work problems typically involve two or more people or machines working on the same task.

Work done = (Rate)(Time)

The key to these problems is to express the rate of work per unit of time, whether an hour, a day, a week, or some other unit.

For example, if a machine can do a job in **n** days, then:

rate of machine = **1/n** job per day

If machine is used for **d** days, it performs **d(1/n)** = **d/n** of the job.

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Work Problem Example

"Using a small mower, at 12 noon a student begins to mow a lawn, a job that would take 9 hours working alone. At 1 P.M. another student, using a tractor, joins the first student and they complete the job together at 3 P.M. How many hours would it take to do the job using only the tractor?"

SOLUTION:

t = number of hours to do the job by tractor alone.

*The small mower works from 12 noon to 3 P.M., or **3 hours**.*

*The tractor is used from 1 P.M. to 3 P.M., or **2 hours**.*

Machine	Rate	Time	Work Done
Mower	**1/9**	***3***	***3/9 = 1/3***
Tractor	**1/t**	***2***	***2 / t***

Work done by small mower + Work done by tractor = 1 Whole job

$$1/3 + 2/t = 1$$

$$(1/3 + 2/t)\,(3t) = 1\,(3t)$$

$$t + 6 = 3t; \quad t = 3$$

Thus Tractor alone can do in 3 hours

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Formula Problems

There are many formulas in mathematics that define the example.

Circumference of a circle in terms of its radius, ***r***:

$$C = 2\pi r$$

Circumference of a circle in terms of its diameter, ***d***:

$$C = \pi d$$

The perimeters and areas of polygons and other shapes also have formulas associated with them.

Other than being given the formula, they are solved in the same way as the problems noted earlier…

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Questions?

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Quick Problems

Ch. 2.2, #54. Write formulas for each of the following:

a. ***"the charge in cents for a telephone call between two cities lasting n minutes, n greater than 3, if the charge for the first 3 minutes is $1.20 and each additional minute costs 33 cents."***

You have 4 minutes to solve this …

$$\textbf{\textit{Charge}} = 120 + (n - 3)(33)$$
$$C = 120 + 33n - 99$$
$$\mathbf{C = 21 + 33n} \quad \textbf{\textit{final answer for all } } n \geq 3$$

Quick Problems

Ch. 2.2, #54. Write formulas for each of the following:

b. "the taxi fare for m miles, if the initial charge is $2.50 and the driver charges 70 cents for every 1/5th mile traveled."

You have 3 minutes to solve this …

$$Fare = \$2.50 + \$0.70(\ m/(1/5)\ )$$
$$F = \$2.50 + \$3.50m \quad \textbf{\textit{final answer}}$$

Quick Problems

Ch. 2.2, #54. Write formulas for each of the following:

c. ***"the amount in an account at the end of a year, if simple interest is paid at the rate of 16%, and the account contains d dollars at the beginning of the year."***

You have 3 minutes to solve this …

$$d + 0.16d = d\,(\,1.00 + 0.16\,)$$
$$\textbf{\textit{Amount in Account} = 1.16d} \quad \textbf{\textit{final answer}}$$

Quick Problems

Ch. 2.2, #54. Write formulas for each of the following:

d. ***"the fine a company paid for dumping acid into the Mississippi River for d days, if the U.S. Environmental Protection Agency fined the company $150,000 plus $1000 per day until the company complied with the federal water pollution regulations."***

You have 3 minutes to solve this …

Fine = $150,000 + $1,000d ***final answer***

Quick Problems

Ch. 2.2, #55. ***"Find three consecutive even numbers such that twice the first plus 3 times the second is 4 times the third."***

You have 4 minutes to solve this …

1st consecutive , even number: x
2nd consecutive, even number: $x + 2$
3rd consecutive, even number: $x + 4$

$$2x + 3(x + 2) = 4(x + 4)$$
$$2x + 3x + 6 = 4x + 16$$
$$5x + 6 = 4x + 16$$
$$x = 10; \quad x + 2 = 12; \quad x + 4 = 14$$

The numbers are 10, 12, and 14 ***final answer***

Quick Problems

Ch. 2.2, #56. ***"When exercising, Mary walks a distance to warm up, jogs 3 ½ times as far as she walks, and sprints 3 1/3 times as far as she jogs. If she covers 4171 meters, find the distances that she walked, jogged, and sprinted."*** *You have 4 minutes to solve this …*

Walking distance $= x$; *Jogging distance* $= \left(3\frac{1}{2}\right)(x) = \frac{7}{2}x$; *and*

Sprinting distance would be: $\left(3\frac{1}{3}\right)\left(\frac{7}{2}x\right) = \left(\frac{10}{3}\right)\left(\frac{7}{2}x\right) = \frac{35}{3}x$

Solving for x: $x + \frac{7}{2}x + \frac{35}{3}x = 4{,}171$

$$6\left[x + \frac{7}{2}x + \frac{35}{3}x\right] = 6(4{,}171)$$
$$6x + 21x + 70x = 25{,}026$$
$$97x = 25{,}026$$
$$x = 258$$

So, she Walks 258m

$$\frac{7}{2}x = 903$$

Jogs 903m and

$$\frac{35}{3}x = 3{,}010$$

Sprints 3,010m

Quick Problems

Ch. 2.2, #57. "*A 10-quart radiator has 30% antifreeze. How much of the fluid should be drained and replaced with pure antifreeze to double the strength of the mixture?*"

You have 3 minutes to solve this …

30% antifreeze: $10 - x$ + 100% antifreeze: x = 2(30%) antifreeze: 10

Thus: $0.30(10-x) + 1(x) = [2(0.30)](10) = 0.6(10)$

$(3 - 0.3x) + x = 6$

$0.7x = 3$

$x = 4.29$

So… 4.29 quarts should be drained and replaced

Questions?

Chapter 2

2.3 The Quadratic Equation
$ax^2 + bx + c = 0$

Solving by Factoring

We can factor the left hand side of the Quadratic Equation

$$ax^2 + bx + c = 0$$

into two linear factors to solve the equation.

So for the quadratic, $x^2 + 5x + 6 = 0$

Factoring into: $(x - 2)(x - 3) = 0$

BUT, this is only true for $(x - 2) = 0$ *or* $(x - 3) = 0$

Thus we have two possible results:

$x = 2$ *AND* $x = 3$

that will satisfy the equation

Common Factors

We can also factor the left hand side of the Quadratic Equation where c=0 :

$$ax^2 - bx = 0$$

into two linear factors to solve the equation.

So for the general case, $x(ax - b) = 0$

$(ax - b) = 0 \,/\, x$

BUT, this is only true for $(ax - b) = 0$

Thus we have ONE possible result:

$x = b/a$

that will satisfy the equation

(x=0 is a useless answer)

Special Cases $x^2 - p = 0$

Some quadratic equations do not necessarily require the use of factoring when finding solutions:

$x^2 - p = 0$ $\quad x^2 + p = 0$

Because of their special form, we may use the method of taking roots.

Where $x^2 - 3 = 0$ $\quad x^2 = 3$ $\quad x = +\sqrt{3},\ x = -\sqrt{3}$

this is often abbreviated: $x = \pm\sqrt{3}$

You might also 'see' this as difference of squares and write:

$(x + \sqrt{3})(x - \sqrt{3}) = 0$

So for the general case, $x^2 - p = 0$

is solved as: $x^2 - p = (x + \sqrt{p})(x - \sqrt{p}) = 0$

for $p > 0$

Special Cases: $x^2 + p = 0$

Where $x^2 + p = 0$

We might 'see' this as difference of squares and write:

$$x^2 + p = (x + \sqrt{pi})(x - \sqrt{pi}) = 0$$

for $p > 0$ *and where* $i = \sqrt{-1}$

$i = \sqrt{-1}$ is a representation of an IMAGINARY NUMBER

However, please NOTE!!!

Imaginary Numbers

are *NOT* part of your CM/FM studies…

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Next Class Session #4

Class Sessions Posted Online Friday before Class

- ***DUE:*** ***Homework Set #03 by 12:00Noon 21 September!***
  NOTE: Quiz 3 = four problems from hwk
- ***Due:*** ***Textbook readings***
- ***Lecture: Solving Equations of Two Variables***
- ***Review:*** ***Homework Set #03***

NO CLASS Monday - 28 September *= Instructor's Holiday*

** Exam #1 Emailed Monday 5 October by 12:00Noon **

In class **– Session 5: Monday 5 October:**

- ***DUE: Exam 1,*** ***Homework Set #04 by 12:00Noon***
  NOTE: Quiz 4 = four problems from hwk
- ***Review:*** ***Textbook readings***
- ***Lecture: Creating Equations – Polynomial Functions***

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Any Questions?

Send me an email …

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End

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