Pratt

Math 150 – Fall 2020

Algebra & Trigonometry

Charles Rubenstein, Ph. D.
Professor of Engineering & Information Science

Session 4: Monday 9/21/20

6:30pm - 9:20pm

via REMOTE LEARNING

Revision 1

Instructor Contact Information

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Fall 2020 Virtual Office hours *ONLY*
Wednesdays 10:00am-2:00pm via Zoom Meeting
To make your appointment
Send me an email at least one day in advance :
c.rubenstein@ieee.org
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This Class Session

Class Sessions Posted Online Friday before Class

- ***DUE:*** ***Homework Set #03 by 12:00Noon 21 September!***
  NOTE: Quiz 3 = four problems from hwk
- ***Due:*** ***Textbook readings***
- ***Lecture: Solving Equations of Two Variables***
- ***Review:*** ***Homework Set #03***

NO CLASSES Monday - 28 September *= Instructor's Holiday*

**** Exam #1 Emailed Monday 5 October by 1:00pm ****
**** Exam #1 DUE Monday 5 October by 5:00pm ****

In class **– Session 5: Monday 5 October:**

- ***DUE:*** ***Exam #1,*** ***Homework Set #04 by 12:00Noon 5 October!***
  NOTE: Quiz 4 = four problems from hwk
- ***Review:*** ***Textbook readings***
- ***Lecture: Creating Equations – Polynomial Functions***

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About the Homework Quizzes

Since there are dozens of practice problems within each homework set, I feel it unnecessary to have you send ALL the problems to me each week.
I have selected four (4) problems from each homework for you to submit and - as long as at least three are answered correctly - receive 'quiz' credit of 3% for correct answers.
These are the selected problems for the next homework assignments:

HWK #04: Section 2.2: 25, 28, 48 and Section 2.3: 16
HWK #05: Section 2.4: 1, 4, 8, 21
HWK #06: Section 2.2: 54a, 54c, 55, 57

Today's Homework was due not later than Noon today.
The other homeworks are due each of the following weeks at Noon.

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About Emailing me your Homework

Homework is due not later than Noon on class days.

1. Use ***DARK, BLACK pencil or pen***. ***Include your Name and Date!***
   If I can't read your work you get a ZERO!
2. Please ***scan your work*** as a PDF and save it as ***lastname_xx.pdf***
   HOWEVER – IF YOU CAN NOT SCAN –
   a. Take photo(s) of your work and insert the photo(s) into a Word (***rtf***, ***doc***, ***docx***) document
   b. Save as ***lastname_xx.docx*** (etc.), where ***xx*** is your assignment number.

Then email your file to me:
c.rubenstein@ieee.org
With the subject line ***Math Homework #XX***.
Email me ONLY the requested four (4) problems.
(Email any you might be challenged by in a separate document)

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About Exam #1 – Worth 20%

Exam #1 is a one hour exam

It will be emailed to you on Monday 5 October by 1:00pm

EXAM 1 is DUE by 5:00pm on 5 October!

See HOW TO EMAIL ME YOUR EXAM on the next slide

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About Emailing me your Exam(s)

Exams will be emailed to you not later than 1:00pm (ET) and must be received not later than 5:00pm that day.

(Exam 1: 5 October; Exam 2: 9 November; Final 7 December)

1. *Use **DARK, BLACK pencil or pen**. **Include your Name & Date***
   If I can't read your work you get a ZERO!
2. *Please **scan your work** as a PDF and save it as **lastname_EE.pdf***

HOWEVER – IF YOU CAN NOT SCAN –

a. *Take photo(s) of your work and insert the photo(s) into a Word (**rtf**, **doc**, **docx**) document*

b. *Save as **lastname_EE.docx** (etc.), where **EE** is your exam 'ID' – **E1**, **E2** or **FE**(final exam).*

Then email your file** to me: **c.rubenstein@ieee.org

*With the subject line **Exam EE**.*

Email me BOTH the worked out problems AND solutions.

Be sure to include any units…

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Math 150 – Class Topics

1. **The Foundations of Algebra**
2. **Equations and Inequalities**
3. **Functions**
4. **Polynomial Functions**
5. **Rational Functions and Conic Sections**
6. **Exponential and Logarithmic Functions**
7. **The Trigonometric Functions**
8. **Analytic Trigonometry**
9. **Applications of Trigonometry**
10. **Systems of Equations and Inequalities**
11. **Matrices, Linear Systems, and Determinants**
12. **Topics in Algebra**

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Draft Schedule: Math 150 – Fall 2020 – Remote Learning

Monday	Notes
24-Aug	1. Introduction: Numbers, Arithmetic Operations, Fractions
31-Aug	2. Manipulation of Algebraic Expressions; *Hwk #1 Due @ Noon*
7-Sep	NO CLASSES – Labor Day
14-Sep	3. Solving Linear and Quadratic Equations of One Variable; *Hwk #2 Due*
21-Sep	4. Solving Equations of Two Variables; *Hwk #3 Due*
28-Sep	NO CLASSES – Instructor Holiday
5-Oct	5. Creating Equations – Polynomial Functions; *Hwk #4 Due; Exam #1*
12-Oct	6. Polynomial Functions, continued; *Hwk #5 Due; Exam 1 Review*
19-Oct	7. Functions, Graphing, Exponents and Logarithms; *Hwk #6 Due*
26-Oct	8. Trigonometric Functions, Pythagorean Theorem; *Hwk #7 Due*
2-Nov	9. Applications of Trigonometry; *Hwk #8 Due*
9-Nov	10. Analytic Trigonometry: Identities & Graphing; *Hwk #9 Due; Exam #2*
16-Nov	11. Areas and Volumes of Geometric Solids; *Hwk #10 Due; Exam 2 Review*
23-Nov	12. Systems of Equations and Inequalities;
30-Nov	13. Series and Sequences, Review topics
7-Dec	Final Examination *Exam Emailed by 12:00Noon - Due at 5:00pm ET*

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www.CharlesRubenstein.com/150

20fa04.pdf = *This slide set**

20fa04_h.pdf = *slides as 6-up handouts**

**My goal is to post these not later than Noon on the Friday one week before our Zoom Class Meetings*

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Homework Quizzes

We will go over the 'Quiz' problems in class and then they will appear in an updated slide set.

Let me know if there are any problems in this homework set that you might have had challenges on and we can review them…

It would be appreciated if you did so when you submitted your four homework problems for the 'quiz'

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Homework #03 Selected Review Problems

*(**NOTE: Section 1.8** Problems 1, 2, 3, 27, 31 = Imaginary Numbers – **NOT REQUIRED**)*

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Homework #3 Review

Ch. 1 Pp 79-82

*#17. "A salesperson receives **3.25x + 0.15y** dollars, where **x** is the number of hours worked and **y** is the number of miles driven. Find the amount due the salesperson if **x = 12 hours** and **y = 80 miles**."*

$3.25x + $0.15y = ?

= $3.25*(12)* + $0.15*(80)*

= $39.00 + $12.00 = **$51.00** ← **ans.**

(Clearly this is a problem that does not take into consideration today's $15+ hourly rates and $0.58 IRS mileage reimbursement…)

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Homework #3 Review

Ch. 1 Pp 79-82

In Exercises 21–23, perform the indicated operations.

#22. $x(2x-1)(x+2)$

$= (2x^2-x)(x+2)$

$= (2x^3-x^2)+(4x^2-2x)$

$= 2x^3+3x^2-2x$

In Exercises 24–29, factor each expression.

#26. $2a^2+3ab+6a+9b$

$= 2a(a+3) + 3b(a+3)$

$= (2a + 3b)(a+3)$

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Homework #3 Review

Ch. 1 Pp 79-82 #78. *The irrational number called the '**golden ratio**': $T = \frac{\sqrt{5}+1}{2}$ has properties that have intrigued artists, philosophers, and mathematicians through the ages.*

Show that **T** satisfies the identity $T = 1 + \frac{1}{T}$

$$\frac{\sqrt{5}+1}{2} = 1 + \frac{1}{\frac{\sqrt{5}+1}{2}}$$

$$1 + \frac{1}{\frac{\sqrt{5}+1}{2}} = 1 + \left[\frac{1}{\frac{\sqrt{5}+1}{2}} \cdot \frac{(2)}{(2)}\right]$$

$$= 1 + \frac{2}{\sqrt{5}+1}$$

$$= 1 + \left[\frac{2}{\sqrt{5}+1} \cdot \frac{\sqrt{5}-1}{\sqrt{5}-1}\right]$$

$$= 1 + \frac{2(\sqrt{5}-1)}{5-1}$$

$$= 1 + \frac{2(\sqrt{5}-1)}{4}$$

$$= \frac{4}{4} + \frac{2(\sqrt{5}-2)}{4}$$

$$= \frac{4+2\sqrt{5}-2}{4}$$

$$= \frac{2+2\sqrt{5}}{4}$$

$$= \frac{2(1+\sqrt{5})}{4}$$

$$= \frac{1+\sqrt{5}}{2}$$

$$= \frac{\sqrt{5}+1}{2} = T$$

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Homework #3 Review

Ch. 2.1 (Linear Equations in One Unknown) Pp 93-94

Problems 5, 6, 7, 13, 25, 26, 31, 33

In Exercises 5–24, solve the given linear equation, check your answer.

#5. $3x+5 = -1$

$3x = -6$ $x = -2$

#6. $5r + 10 = 0$

$5r = -10$ $r = -2$

#7. $2 = 3x + 4$

$(2 - 4) = 3x$ $-2 = 3x$ $-2/3 = x$

#13. $-5x + 8 = 3x - 4$

$-5x - 3x = -8 - 4$ → $-8x = -12$ $x = 3/2$

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Homework #3 Review

Solve for x in Exercises 25–28.

#25. $kx + 8 = 5x$

$8 = 5x - kx$ → $8 = x(5 - k)$

$8/(5 - k) = x$; *for $k \neq 5$*

#26. $8 - 2kx = -3x$

$8 = 2kx - 3x$ → $8 = x(2k - 3)$

$8/(2k - 3) = x$ *for $k \neq 3/2$*

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Homework #3 Review

Solve and check in Exercises 29–44.

#31. $\frac{2}{x}+1=\frac{3}{x}$ $\left(\frac{2}{x}+1\right)(x) = \left(\frac{3}{x}\right)(x)$

$2 + x = 3$

$x = 1$

#33. $\frac{2y-3}{y+3}=\frac{5}{7}$ $\left(\frac{2y-3}{y+3}\right)[7(y+3)] = \left(\frac{5}{7}\right)[7(y+3)]$

$7(2y-3) = 5(y+3)$

$14y - 21 = 5y + 15$

$9y = 36$

$y = 4$

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Homework #3 Review

Ch. 2.2 *Pp 103-104 Problems 1, 2, 3, 4, 5, 7, 11, 12*

In Exercises 1–3, ***let n represent the unknown****. Translate from words to an algebraic expression or equation.*

#1. The number of blue chips is **3** more than **twice** the number of red chips.

Number of red chips: ***n*** **Number of blue chips:** ***3 + 2n***

#2. The number of station wagons on a parking lot is **20** fewer than **3** times the number of sedans.

Number of sedans: ***n*** **Number of station wagons:** ***3n – 20***

#3. **Five less than 6 times** a number is **26**.

$(6n - 5) = 26$

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Homework #3 Review

In Exercises 4–41, translate from words to an algebraic problem and solve.

#4. Janis is 3 years older than her sister. Thirty years from now the sum of their ages will be 111. Find the current ages of the sisters.

Who?	Age Now	Age in 30 years
Janis	*x + 3*	*(x + 3) + 30*
Sister	*x*	*x + 30*

[(x + 3) + 30] + (x + 30) = 111

2x + 63 = 111 thus **2x = 48** and

x = 24 = sister's age

x + 3 = 27 = Janis' age

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Homework #3 Review

In Exercises 4–41, translate from words to an algebraic problem and solve.

#5. John is presently 12 years older than Fred. Four years ago John was twice as old as Fred. How old is each now?

Who?	Age Now	Age 4 years ago
Fred	*x*	*x - 4*
John	*x+12*	*(x + 12) - 4*

(x + 12) – 4 = 2(x – 4)

x + 8 = 2x – 8 *and thus*

x = 16 = Fred's age

x + 12 = 28 = John's age

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Homework #3 Review

In Exercises 4–41, translate from words to an algebraic problem and solve.

#7. Find three consecutive integers whose sum is 21

1st number: x

2nd number: x + 1

3rd number: x + 2

(x) + (x + 1) + (x + 2) = 21

3x + 3 = 21 → 3x = 18

and **x = 6**; thus **x + 1 = 7**; **x + 2 = 8**

The three consecutive numbers are 6, 7, and 8.

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Homework #3 Review

#11. A 12-meter long steel beam is to be cut into two pieces so that one piece will be 4 meters longer than the other. How long will each piece be?

Length of 1st piece: **x** Length of 2nd piece: **x + 4**

(x) + (x + 4) = 12

2x + 4 = 12 → 2x = 8 thus; **x = 4** and **x + 4 = 8**

The lengths are **4m** and **8m**.

#12. A rectangular field whose length is 10 meters longer than its width is to be enclosed with exactly 100 meters of fencing material. What are the dimensions of the field?

Width: **w** Length: **w + 10**

2(w) + 2(w + 10) = 100

4w + 20 = 100 → 4w = 80 → thus **w = 20**

therefore, **w + 10 = 30**

The width is **20m** and the length is **30m**

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Questions?

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Topics in Session 3

Ch. 1 The Foundations of Algebra

1.8 Complex Numbers

(NOTE: There are normally NO complex numbers in CM/FM problems. Complex numbers DO exist in electrical and mechanical engineering…)

Chapter 1 Review

Ch. 2 Equations and Inequalities

2.1 Linear Equations in One Unknown

2.2 Applications: From Words to Algebra

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In Chapter 2

Equations and Inequalities

2.1 Linear Equations in One Unknown

2.2 Applications: From Words to Algebra

2.3 The Quadratic Equation

2.4 Applications of Quadratic Equations

2.5 Linear and Quadratic Inequalities

2.6 Absolute Value in Equations and Inequalities

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Chapter 2

2.3 The Quadratic Equation

$ax^2 + bx + c = 0$

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REVIEW: Solving by Factoring

We can factor the left hand side of the Quadratic Equation

$ax^2 + bx + c = 0$

into two linear factors to solve the equation.

So for the quadratic, $x^2 + 5x + 6 = 0$

Factoring into: $(x + 2)(x + 3) = 0$

This is only true for $(x + 2) = 0$ *or* $(x + 3) = 0$

Thus we have two possible results:

$x = -2$ *AND* $x = -3$

that will satisfy the equation

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Quick Problems

Ch. 2, Pg 109 (5^{th} =100) #a

Solve the equation: $4x^2 - x = 0$

You have 3 minutes to solve this

$4x^2 - x = 0 \rightarrow x(4x - 1) = 0$

Resist the temptation to solve by dividing by x as the solution requires TWO values of x :

$x = 0$* and $x = ¼$ ***final answer***

**NOTE:* $x = 0$ may not always be realistic, but might be possible

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Quick Problems

Now solve the equation: $4x^2 - 4x = 0$

You have 2 minutes to solve this

$4x^2 - 4x = 0 \rightarrow 4x(x - 1) = 0$

Resisting the temptation to solve by dividing by x as the solution requires TWO values of x :

$4x = 0 \rightarrow x = 0$

$x - 1 = 0 \rightarrow x = 1$

$x = 0$* and $x = 1$ ***final answer***

**NOTE: x = 0 may not always be realistic, but might be possible*

Special Cases: $a(x+h)^2 + c = 0$

Equations of the form $a(x+h)^2 + c = 0$

May be solved in a straightforward manner:

$a(x+h)^2 + c = 0$

$a(x+h)^2 = -c$

$(x+h)^2 = -c/a$

$(x+h) = \pm\sqrt{-c/a}$

and thus $x = -h \pm\sqrt{-c/a}$

NOTE: Only valid for CM/FM if ***c*** *or* ***a*** *are negative making* $-c/a$ *positive – we do not use complex numbers*

Special Case Example

Example 7 (Pg 112; 5th 102)

Where $2(x-1)^2 - 6 = 0$

We solve in a straightforward manner:

$2(x-1)^2 = 6$

$(x-1)^2 = 3$

$(x-1) = \pm\sqrt{3}$

and thus $x = 1 \pm\sqrt{3}$

Quick Problems

Ch. 2, Pg 113 (5th =103) #b

Solve the equation: $(2x - 7)^2 - 5 = 0$

You have 3 minutes to solve this …

$(2x - 7)^2 - 5 = 0 \rightarrow (2x - 7)^2 = 5$

$(2x - 7) = \pm\sqrt{5}$

$2x = 7 \pm\sqrt{5}$

$x = \dfrac{7 \pm\sqrt{5}}{2}$ ***final answer***

Completing the Square *(optional)*

ANY quadratic equation can be rewritten in the form

$a(x+h)^2 + c = 0$

*By the technique of "**completing the square**"*

If we start with: x^2+dx *we are missing the constant* h^2

such that we have: $x^2 + dx + h^2 = (x+h)^2$

Expanding the right side: $x^2 + dx + h^2 = x^2 + 2hx + h^2$

Therefore $dx = 2hx$

such that $h = d/2$

and $h^2 = (d/2)^2$

To complete the square add $h^2 = (d/2)^2$ *to* x^2+dx

Yielding the equation: $x^2 + dx + (d/2)^2 = 0$

Completing the Square Example

Example 10 (Pg 114; 5th 104)

Solve by completing the square: $2x^2 - 10x + 1 = 0$

In a straightforward manner:

$2x^2 - 10x = -1$ *therefore* $2(x^2 - 5x) = -1$

$x^2 - 5x = -\frac{1}{2}$ *eq. 1*

$h = d/2; h = -5/2 \rightarrow h^2 = (d/2)^2; h^2 = 25/4$

Adding to both sides of equation 1: $x^2 - 5x + 25/4 = -\frac{1}{2} + 25/4$

$(x - 5/2)^2 = 23/4$ *thus* $x - 5/2 = \pm(\sqrt{23}/4)$

Such that $x = 5/2 \pm(\sqrt{23})/2$

and finally $x = \frac{1}{2}(5 \pm \sqrt{23})$

The Quadratic Formula

Once we write the quadratic equation in the form

$ax^2 + bx + c = 0$ *for* $a \neq 0$

We can solve the equation in the most general terms

To find the general expression for the two roots of the equation:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

NOTE!!! This is NOT reducible to:

$$x \neq -b \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

Please be careful if you use your calculator to solve these equations as it is easy to make entry errors…

Quadratic Formula Example

Example 11 (Pg 115; 5th 105)

Solve: $2x^2 - 3x - 3 = 0$ *using the quadratic formula*

In a straightforward manner: $a = 2, b = -3, c = -3$

Substitute into the formula; $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Such that $x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(-3)}}{2(2)}$

$$x = \frac{+3 \pm \sqrt{(9 + 24)}}{4}$$

$$x = \frac{(3 \pm \sqrt{33})}{4}$$

and finally $x = \frac{1}{4}(3 \pm \sqrt{33})$

The Descriminant: $b^2 - 4ac$

The roots of the quadratic equation in the form

$ax^2 + bx + c = 0$ *for* $a \neq 0$

may be solved using the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Where $b^2 - 4ac$ *is called the* ***descriminant:***

When $b^2 - 4ac$ *is positive* → *roots are real numbers*

When $b^2 - 4ac$ *is zero* → *there is a double root*

When the descriminant result is a square, the roots are rational

When $b^2 - 4ac$ *is negative* → *roots are complex conjugate pairs*

We do not use complex numbers in CM/FM…

Determinant Examples

DO NOT SOLVE, express nature of the roots for the following equations:

$2x^2 - 3x - 3 = 0$

$b^2 - 4ac$ *is* $(3)^2 - 4(2)(-3) = 9 + 24 =$ **+ 33** = *not a square*

Roots = two real numbers

(Pg 119; 5th 109) #a.

$4x^2 - 20x + 25 = 0$

$b^2 - 4ac$ *is* $(-20)^2 - 4(4)(25) = 400 - 400 = 0$

Roots = real double roots

Descriminant Examples

DO NOT SOLVE, express nature of the roots for the following equations:

(Pg 119; 5th 109) #b

$5x^2 - 6x = -2$

$b^2 - 4ac$ *is* $(-6)^2 - 4(5)(+2) = 36 - 40 =$ **- 4** = *square*

Roots = complex conjugate pair roots

(Pg 119; 5th 109) #c

$10x^2 = x + 2$

$b^2 - 4ac$ *is* $(-1)^2 - 4(10)(-2) = 1 + 80 =$ **+81** = *square*

Roots = two real, rational roots

Radical Equations

One form of equation that leads to the quadratic equation is the ***Radical Equation*** *of the form:*

$P = Q$ *which is a subset of the solution set of*

$P^n = Q^n$ *for* **n** *a natural number*

Solutions of these equations MUST be checked to confirm that they satisfy the original equations…

If one solution is NOT a solution of the original equation it is called an ***Extraneous Solution****.*

Questions?

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Chapter 2

2.4 Applications of Quadratic Equations

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Word Problems

Word problems can also lead to solutions by the quadratic equation;

"The larger of two positive numbers exceeds the smaller by 2. If the sum of the squares of the two numbers is 74, find the two numbers."

So: x = *larger number;* $(x-2)$ = *smaller number*

Sum of the squares = 74 *and thus the equation is:*

$$(x)^2 + (x-2)^2 = 74$$

And, as shown in the text, $x = 7$ *and* $x = -5$

BUT, -5 *is not a solution as* $x-2 = -7$; $(7)^2 + (-7)^2 \neq 74$

Therefore $x = 7$, ***and the smaller # is 5***

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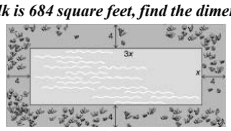
Quick Problems

Ch. 2.4, Example 2. ***"The length of a pool is 3 times its width, and the pool is surrounded by a grass walk 4 feet wide. If the total area covered and enclosed by the walk is 684 square feet, find the dimensions of the pool."***

$A = l \times w = 684$



$$(3w + 8)(w + 8) = 684$$

$$\text{FOIL: } 3w^2 + 64 + 8w + 24w = 684$$

$$\text{As } 684-64 = 620: \quad 3w^2 + 32w - 620 = 0$$

$$\text{Factoring: } (3w + 62)(w - 10) = 0$$

$3w = -62$ *yields* $w = -20.67$ *can't be negative, so…*

$w = 10$; *Pool is 10 feet wide x 30 feet long* ← *ans*

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Quick Problems

Ch. 2.4, Example 3. ***"Working together, two cranes can unload a ship in 4 hours. The slower crane, working alone, requires 6 hours more than the faster crane to do the job. How long does it take each crane to do the job by itself?"***

x = *# hours/job for crane 1*

$x + 6$ = *# hours/job for crane 2*

$$4(1/x) + 4(1/(x+6)) = 1$$

$$4(x+6) + 4x = x^2 + 6x$$

$$0 = x^2 - 2x - 24$$

$$0 = (x+4)(x-6)$$

$x = -4$ *rejected;* $x = 6$

Fast crane does in 6 hours what Slow crane does in 12 ← ***ans***

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Questions?

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In Chapter 3

Functions
3.1 The Rectangular Coordinate System
3.2 Functions and Function Notation
3.3 Graphs of Functions
3.4 Linear Functions
3.5 The Algebra of Functions; Inverse Functions
3.6 Direct and Inverse Variation

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Chapter 3

3.1 The Rectangular Coordinate System

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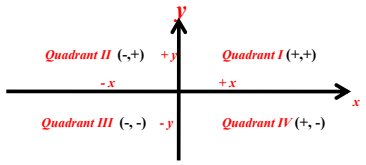
Rectilinear Coordinates

The x-y Coordinate System



Where would you expect to find the following (x, y) points and Which Quadrant are they in?

A: (0,0) B:(1, 1) C: (1, -1) D: (-1, 1) E: (-1, -1)
F: (10,0) G:(0, 5) H: (5, -5) I: (-5, 5) J: (-2, -2)

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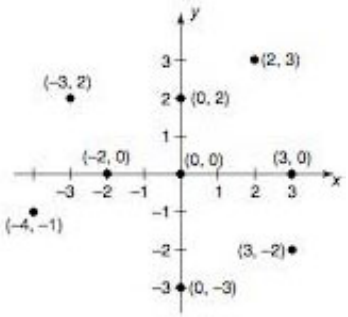
Rectilinear Coordinates

Locating points on the x-y Coordinate System



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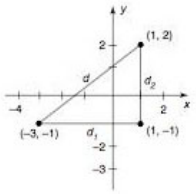
The Distance Formula

The Distance between two points: (-3, -1) and (1, 2)



Create a line parallel to the x axis from point (-3, -1) and another line parallel to the y axis from point (1, 2). In this case, the lines meet at (1, -1). Line $d_1 = 3+1 = 4$ and line $d_2 = 2+1 = 3$. This figure is a right triangle. Therefore the hypotenuse 'd' is defined by: $d^2 = d_1^2 + d_2^2$ and $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Here we find $d = \sqrt{25} = 5$

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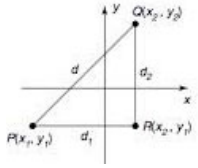
The General Distance Formula

In general the Distance between two points $p = (x_1, y_1)$ and $q = (x_2, y_2)$ is found:



$d_1 = (x_2 - x_1)$ *and* $d_2 = (y_2 - y_1)$

$d^2 = d_1^2 + d_2^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$

Therefore the Pythagorean hypotenuse 'd' is defined by:

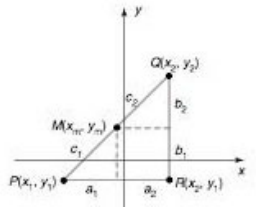
$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

When this equation is correct, the figure is a triangle…

 54

The Midpoint Formula

You can find the midpoint M of any line segment…



If we find the lengths of the lines: $d_1 = (x_2 - x_1)$ & $d_2 = (y_2 - y_1)$

All we need to do is divide these by '2' to find the average line length or midpoint.

In general: $x_m = (x_2 - x_1) / 2$

and similarly $y_m = (y_2 - y_1) / 2$

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Questions?

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Chapter 3

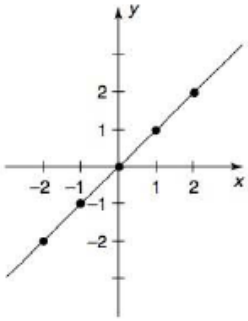
3.3 Graphs of Functions

 57

f(x) = x Identity Function

y = x (increasing)

x	y
−2	−2
−1	−1
0	0
1	1
2	2

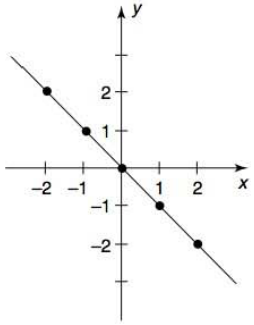


 58

f(x) = -x Negation Function

y = -x (decreasing)

x	y
−2	2
−1	1
0	0
1	−1
2	−2

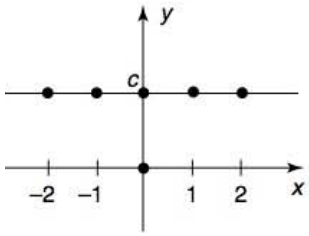


 59

f(x) = c Constant Function

y = c (constant)

x	y
−2	c
−1	c
0	c
1	c
2	c

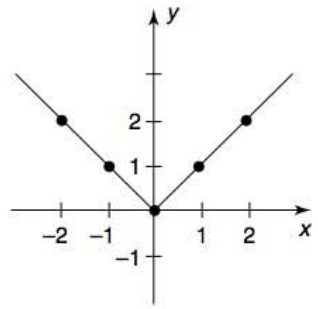


 60

$f(x) = |x|$ Absolute Value Function

$y = |x|$ *(decreasing, then increasing)*

x	y
−2	2
−1	1
0	0
1	1
2	2



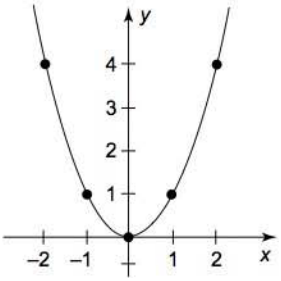
 61

$f(x) = x^2$ Squaring Function

$y = x^2$ *(decreasing, then increasing)*

$y = x^2$ *= a Parabola*

x	y
−2	4
−1	1
0	0
1	1
2	4

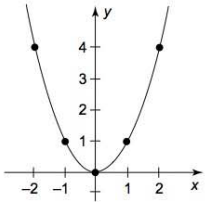


 62

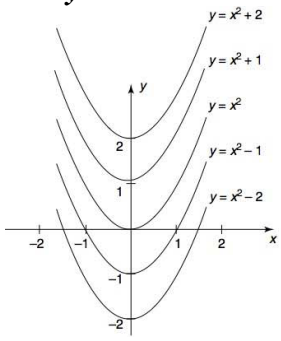
Vertical Shifting of $f(x) = x^2 \pm c$

$y = x^2 \pm c$

Parabolas



$y = x^2$

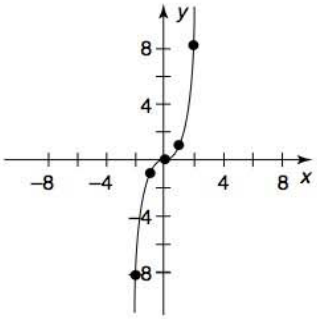


 63

$f(x) = x^3$ Cubing Function

$y = x^3$

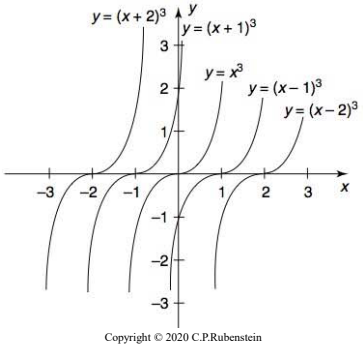
x	y
−2	−8
−1	−1
0	0
1	1
2	8



 64

$f(x) = (x \pm c)^3$ Horizontal Shifts

$y = (x \pm c)^3$ *(cubic or cubing function)*

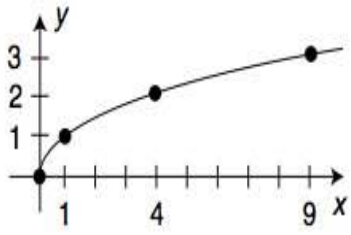


 65

$f(x) = \sqrt{x}$ Square Root Function

$y = \sqrt{x}$

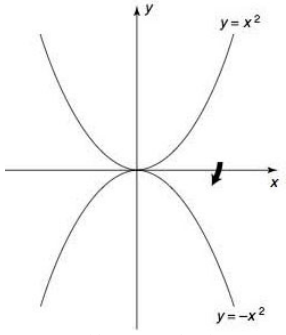
x	y
0	0
1	1
4	2
9	3



 66

$f(x) = \pm x^2$ Reflections…

$y = \pm x^2$



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Shifting Graphs…

Form	Relationship to the Graph of $y = f(x)$, $p > 0$
$y = f(x) + p$	Shift $f(x)$ p units up.
$y = f(x) - p$	Shift $f(x)$ p units down.
$y = f(x - p)$	Shift $f(x)$ p units to the right.
$y = f(x + p)$	Shift $f(x)$ p units to the left.
$y = -f(x)$	Reflect $f(x)$ about the x-axis.
$y = f(-x)$	Reflect $f(x)$ about the y-axis.

Table 2, Page 188

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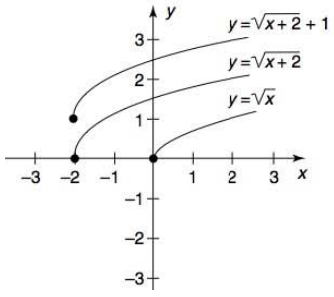
Quick Problem…

Sketch the graph of $y = \sqrt{(x + 2)} + 1$

NOTE: This is the graph $y = \sqrt{x}$ shifted two units to the left, and then shifted one unit up.



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Word Problem…

The commission earned by a door-to-door cosmetics salesperson is determined as shown in the accompanying table.

a. Express the commission **C** as a function of sales **s**.
b. Find the commission if the weekly sales are $425.
c. Sketch the graph of the function.

Weekly Sales	Commission
less than $300	20% of sales
$300 or more but less than $400	$60 + 40% of sales over $300
$400 or more	$100 + 60% of sales over $400

A. The function C can be described by three equations:

$$C(s) = \begin{cases} 0.20s & \text{if } 0 \le s < 300 \\ 60 + 0.40(s - 300) & \text{if } 300 \le s < 400 \\ 100 + 0.60(s - 400) & \text{if } s \ge 400 \end{cases}$$

*B. When **s = 425**, use the third equation:*

$$C(425) = 100 + 0.60\,(425 - 400) = 100 + 0.60(25) = 115$$

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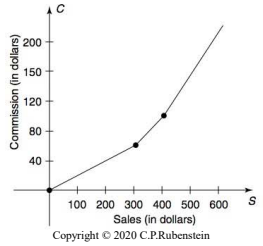
Word Problem *continued*

C. Sketch the graph of the function.

The function C can be plotted using the three equations

$$C(s) = \begin{cases} 0.20s & \text{if } 0 \le s < 300 \\ 60 + 0.40(s - 300) & \text{if } 300 \le s < 400 \\ 100 + 0.60(s - 400) & \text{if } s \ge 400 \end{cases}$$

*where **s=0**, **C = 0**; when **s=300**, **C = 60**; and when **s=425**, **C = 115**:*



 71

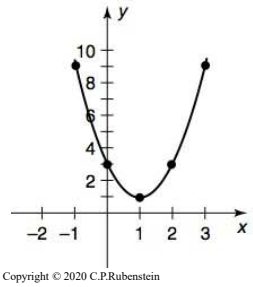
$f(x) = ax^2 + bx + c$ Polynomial Functions

$y = ax^2 + bx + c$ (quadratic function) $a \neq 0$

Let's Try $y = 2x^2 - 4x + 3$

For x = -1: y = 2(+1) – 4(-1) + 3 = 2+4+3 = 9

x	y
−1	9
0	3
1	1
2	3
3	9



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Questions?

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Chapter 3

3.4 Linear Functions

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The Slope of a Line

The polynomial $f(x) = ax + b$ is a linear function and when graphed, will show a straight line…

We can readily show that if this line is not vertical or horizontal, moving from one point on the line to another can be calculated using the differences in the x's and y's in a ratio called the slope of the line "m" where:

$$m = (y - y_1) / (x - x_1)$$

When ***m>0,*** *the line graph is an* ***increasing function***

When ***m<0,*** *the line graph is a* ***decreasing function***

When ***m=0,*** *the line graph is a* ***constant function***

When ***the line is vertical 'm' does not exist = no function!***

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The Point - Slope Formula

We can plot a straight line using the relationship between any two points on that line:

$$m = (y - y_1) / (x - x_1)$$

We can rewrite this as:

$$(y - y_1) = m (x - x_1)$$

This is the Point-Slope formula that defines an equation of a line with slope m that passes through the point (x_1 , y_1)

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The Slope - Intercept Formula

We can also plot a straight line using the more common relationship between any two points on that line:

$$y = m x + b$$

This formula defines an equation of a line with slope m and y-intercept b

Note that these formulas do not define <u>horizontal</u> lines, when m = 0:

$$y = b$$

nor <u>vertical</u> lines where a = the x-intercept:

$$x = a$$

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Questions?

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From Words to Algebra

1. Read the problem through the first time to get a general idea of what is being asked.
2. Read the problem a second time to recognize what may be important in determining that which is to be found.
3. If possible, estimate the solution to this problem, and then compare this estimate with your final answer.
4. Let some algebraic symbol denote the quantity to be found.
5. If possible, represent other quantities in the problem in terms of the algebraic symbol designated in Step 4.
6. Find various relationships (equations or inequalities) in the problem.
7. Use relationships established in Step 6 to find the solution to the problem.
8. Verify that your answer is, indeed, the solution to the problem.

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Questions?

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Next Class Session #5

NO CLASSES Monday - 28 September = Instructor's Holiday
Class Sessions Posted Online Friday before Class

****** Exam #1 Emailed Monday 5 October by 1:00pm ******

- ***DUE:*** ***Exam #1*** ***by 5:00pm 5 October!***
- ***Due:*** ***Homework Set #04***
- ***Due:*** ***Textbook readings***
- ***Lecture: Creating Equations – Polynomial Functions***
- ***Review:*** ***Homework Set #04***

In class **– Session 6: Monday 12 October:**

- ***DUE:*** ***Homework Set #05 by 12:00Noon 5 October!***
- ***Review:*** ***Textbook readings***
- ***Lecture: Polynomial Functions, continued***
- ***Review: Exam 1,*** ***Homework Set #05***

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Any Questions?
Send me an email …

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or
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End

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