Pratt

Math 150 – Fall 2020
Algebra & Trigonometry

Charles Rubenstein, Ph. D.
Professor of Engineering & Information Science

Session 5: Monday 10/05/20
6:30pm - 9:20pm
via REMOTE LEARNING
Revision 1

Instructor Contact Information

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Fall 2020 Virtual Office hours *ONLY*
Wednesdays 10:00am-2:00pm via Zoom Meeting
To make your appointment
Send me an email at least one day in advance :
crubenst@pratt.edu
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This Class Session #05

Class Sessions Posted Online Friday before Class

In class – Session 5: Monday 5 October:

- ***DUE:** Homework Set #04 by 12:00Noon 5 October!*
- *NOTE: Quiz 4 = four problems from hwk*
- ***Review:** Homework Set #04; Textbook readings*
- ***Lecture: Creating Equations – Polynomial Functions, Exponential and Logarithmic Functions***

In class – Session 6: Monday 12 October:

- ***DUE:** Homework Set #05 by 12:00Noon 12 October!*
- *NOTE: Quiz 5 = four problems from hwk*
- ***Review:** Homework Set #05; Textbook readings*
- ***Lecture: Exponential and Logarithmic Functions, continued***

*** Exam #1 to be Emailed to all immediately after class ****
Exam 1 MUST BE RETURNED VIA EMAIL by 11:00pm

About the Homework Quizzes

I have selected four (4) problems from each homework for you to submit and - as long as at least three are answered correctly - receive 'quiz' credit of 3% for correct answers.

These are the selected problems for the remaining homework assignments:

HWK #05: Section 2.4: 1, 4, 8, 21
HWK #06: Section 2.2: 54a, 54c, 55, 57
HWK #07: Section 3.1: 10, 60 and Section 3.4: 10, 16
HWK #08: Section 1.4: 82c and Section 1.5: 42, 54, 60
HWL #09: Number: 2, 4, 6, 8
HWK #10: Section 7.8: 4, 8, 10, 14

Homework is due not later than 12:00pm Noon ET on day of our class session.
If not emailed by then, a zero grade will be entered

Emailing me your Homework

As noted, I have selected four (4) problems from each homework for you to submit each week per the previous slide.

Homework is due not later than Noon class days.

HOW TO PREPARE YOUR ASSIGNMENT:

1. *Use DARK pencil or pen.*
   If I can't read your work you get a ZERO!
2. *Please scan your work as a PDF and save it as* ***lastname_xx.pdf***

HOWEVER – IF YOU CAN NOT SCAN –

a. *Take a photo of your work*
b. *Insert the photo(s) into a Word document*
c. *Save as* ***lastname_xx.docx*** *or* ***lastname_xx.pdf***

Then email your file to me: ***crubenst@pratt.edu***
Email me ONLY the requested four (4) problems.
(Email any you might be challenged by in a separate document)

About Exam #1 – Worth 20%

Exam #1 is a one hour exam with 7 @ 10 point and 6 @ 5 point questions that will be emailed after our Zoom Class on Monday 12 October (by 8:00pmET)
EXAM 1 is DUE by 11:00pm 12 October!

HOW TO EMAIL ME YOUR EXAM:

1. *You MUST use DARK BLACK pencil or pen on white paper.*
   If I can't read your work you get a ZERO!
2. *Please scan your work as a PDF and save it as* ***lastname_E1.pdf***

HOWEVER – IF YOU CAN NOT SCAN –

Fill out the docx file. Take a photo of any work unable to be 'typed out' and insert the photo(s) into the space allotted and save the file as: ***lastname_E1.docx*** *and attach the file (**NO CLOUD LINKS**)*
Include the worked out problems AND solutions AND any units…

Email your file to me at: ***crubenst@pratt.edu***
With the Subject Line: ***Math150 Exam***

Math 150 – Class Topics

1. The Foundations of Algebra
2. Equations and Inequalities
3. Functions
4. Polynomial Functions
5. Rational Functions and Conic Sections
6. Exponential and Logarithmic Functions
7. The Trigonometric Functions
8. Analytic Trigonometry
9. Applications of Trigonometry
10. Systems of Equations and Inequalities
11. Matrices, Linear Systems, and Determinants
12. Topics in Algebra

Draft Schedule: Math 150 – Fall 2020 – Remote Learning

Monday	Notes
24-Aug	1. Introduction: Numbers, Arithmetic Operations, Fractions
31-Aug	2. Manipulation of Algebraic Expressions; *Hwk #1 Due @ Noon*
7-Sep	NO CLASSES – Labor Day
14-Sep	3. Solving Linear and Quadratic Equations of One Variable; *Hwk #2 Due*
21-Sep	4. Solving Equations of Two Variables; *Hwk #3 Due*
28-Sep	NO CLASSES – Instructor Holiday
5-Oct	5. Creating Equations: Polynomials, Exponents & Logarithms *Hwk #4 Due*
12-Oct	6. Exponents and Logarithms, Continued; *Hwk #5; Exam #1 – 'in class'*
19-Oct	7. Functions, Graphing, Exponents and Logarithms; *Hwk #6; Exam Review*
26-Oct	8. Trigonometric Functions, Pythagorean Theorem; *Hwk #7 Due*
2-Nov	9. Applications of Trigonometry; *Hwk #8 Due*
9-Nov	10. Analytic Trigonometry: Identities & Graphing; *Hwk #9 Due; Exam #2*
16-Nov	11. Areas and Volumes of Geometric Solids; *Hwk #10; Exam Review*
23-Nov	12. Systems of Equations and Inequalities
30-Nov	13. Series and Sequences, Review topics
7-Dec	Final Examination *Exam Emailed Monday 9:00am - Due at 1:00pm ET ?*

www.CharlesRubenstein.com/150

HWK7through12.pdf
= *Remaining Homework assignments*

20fa05.pdf = *This slide set**

20fa05_h.pdf = *slides as 6-up handouts**

**My goal is to post these not later than Noon on the Friday one week before our Zoom Class Meetings*

Questions?

Homework #04 Selected Section 2.2 and 2.3 Review Problems

Homework #4 Review

2.2 #25. **Pg 105 (96)** ***"Professors Roberts and Jones, who live 676 miles apart, are exchanging houses and jobs for the summer. They start out for their new locations at exactly the same time, and they meet after 6.5 hours of driving. If their average speeds differ by 4 mph, what are their average speeds?"***

	rate ·	time =	distance
1st prof	x	6.5	$6.5x$
2nd prof	$x + 4$	6.5	$6.5(x + 4)$

$$6.5\,x + 6.5\,(x + 4) = 676$$
$$6.5\,x + 6.5\,x + 26 = 676$$
$$13\,x = 676 - 26 = 650$$
$$x = 50 \text{ and thus } (x + 4) = 54$$

Their speeds are 50 and 54 mph. ← ans.

Homework #4 Review

2.2 #28. **Pg 105 (96)** ***"How many pounds of raisins worth \$3 per pound must be mixed with 10 pounds of peanuts worth \$2.40 per pound to produce a mixture worth \$2.80 per pound? "***

Pounds of raisins: x

$$3x + 2.40(10) = 2.80(10 + x)$$

$$10[3x + 2.40(10)] = 10[2.80(10 + x)]$$ □ ***removes decimal places***

$$30x + 240 = 28(10 + x)$$

$$30x + 240 = 280 + 28x$$

$$x = 20$$

There should be 20 pounds of raisins in the mixture. ans.

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Homework #4 Review

2.2 #34. **Pg 105 (96)** ***"If 1/3 is subtracted from 3 times the reciprocal of a certain number, the result is 25/6. Find the number.?"***

If the number is x:

$$3\left(\frac{1}{x}\right) - \frac{1}{3} = \frac{25}{6}$$

$$6x\left[\frac{3}{x} - \frac{1}{3}\right] = 6x\left(\frac{25}{6}\right)$$

$$18 - 2x = 25x$$

$$18 = 27x$$

x = 2/3 ans.

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Homework #4 Review

2.2 ***In Exercises 42–51 solve for the indicated variable in terms of the remaining variables.***

Pg 105 (96) #42. ***A=Pr solve for r***

$$\frac{1}{P}(A) = \frac{1}{P}(Pr)$$

$$\frac{A}{P} = r$$

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Homework #4 Review

2.2 ***In Exercises 42–51 solve for the indicated variable in terms of the remaining variables.***

Pg 105 (96) #43. ***C=2πr solve for r***

$$\frac{1}{2\pi}(C) = \frac{1}{2\pi}(2\pi r)$$

$$\frac{C}{2\pi} = r$$

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Homework #4 Review

2.2 ***In Exercises 42–51 solve for the indicated variable in terms of the remaining variables.***

Pg 105 (96) #44. ***$V=1/3\ (\pi r^2 h)$ solve for h***

$$\frac{3}{\pi r^2}(V) = \frac{3}{\pi r^2}\left[\frac{1}{3}\pi r^2 h\right]$$

$$\frac{3V}{\pi r^2} = h$$

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Homework #4 Review

2.2 ***In Exercises 42–51 solve for the indicated variable in terms of the remaining variables.***

Pg 105 (96) #45. ***F= 9/5 (C) + 32 solve for C***

$$F - 32 = \frac{9}{5}C$$

$$\frac{5}{9}(F - 32) = C$$

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Homework #4 Review

2.2 ***In Exercises 42–51 solve for the indicated variable in terms of the remaining variables.***

Pg 105 (96) #46. ***S= ½ gt² + vt*** ***solve for v***

$$S - \frac{1}{2}gt^2 = vt$$

$$\frac{1}{t}\left(S - \frac{1}{2}gt^2\right) = \frac{1}{t}(vt)$$

$$\frac{S}{t} - \frac{1}{2}gt = v$$

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Homework #4 Review

2.2 ***In Exercises 42–51 solve for the indicated variable in terms of the remaining variables.***

Pg 105 (96) #47. ***A= ½ h(b+b')*** ***solve for b***

$$\frac{2}{h}(A) = \frac{2}{h}\left[\frac{1}{2}h(b+b')\right]$$

$$\frac{2A}{h} = b + b'$$

$$\frac{2A}{h} - b' = b$$

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Homework #4 Review

2.2 ***In Exercises 42–51 solve for the indicated variable in terms of the remaining variables.***

Pg 105 (96) #48. ***A=P(1+rt)*** ***solve for r***

$$\frac{A}{P} = 1 + rt$$

$$\frac{A}{P} - 1 = rt$$

$$\frac{A}{Pt} - \frac{1}{t} = r$$

OR

$$\frac{A-P}{Pt} = r$$

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Homework #4 Review

2.2 ***In Exercises 42–51 solve for the indicated variable in terms of the remaining variables.***

Pg 105 (96) #49. ***$1/f = 1/f_1 + 1/f_2$*** ***solve for f_2***

$$\frac{1}{f} - \frac{1}{f_1} = \frac{1}{f_2}$$

$$\frac{f_1 - f}{ff_1} = \frac{1}{f_2}$$

Take reciprocal of both sides.

$$\frac{ff_1}{f_1 - f} = f_2$$

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Homework #4 Review

2.3 **In Exercises 1–14, solve by factoring.**

Pg 122 (113) #3. ***$x^2 + x - 2 = 0$***

$$(x + 2)(x - 1) = 0$$

$$(x + 2) = 0 \text{ or } (x - 1) = 0$$

$$x = -2 \text{ or } x = 1$$

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Homework #4 Review

2.3 **In Exercises 15–24, solve the given equation.**

Pg 122 (113) #16. ***$4x^2 - 64 = 0$***

$$4x^2 = 64$$

$$x^2 = 16$$

$$x = \pm 4$$

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Homework #4 Review

2.3 **In Exercises 15–24, solve the given equation.**

Pg 122 (113) #18. $6x^2 - 12 = 0$

$$6x^2 = 12$$
$$x^2 = 2$$
$$x = \pm\sqrt{2}$$

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Homework #4 Review

2.3 **In Exercises 25–36, solve by completing the square.**

Pg 122 (113) #25. $x^2 - 2x = 8$

$$\left[\frac{1}{2}(2)\right]^2 = 1$$
$$x^2 - 2x + 1 = 8 + 1$$
$$(x - 1)^2 = 9$$
$$x - 1 = \pm 3$$
$$x = 1 \pm 3$$
$$x = -2 \text{ or } x = 4$$

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Homework #4 Review

2.3 **In Exercises 25–36, solve by completing the square.**

Pg 122 (113) #26. $t^2 - 2t = 15$

$$\left[\frac{1}{2}(2)\right]^2 = 1$$
$$t^2 - 2t + 1 = 15 + 1$$
$$(t - 1)^2 = 16$$
$$t - 1 = \pm 4$$
$$t = 1 \pm 4$$
$$t = -3 \text{ or } t = 5$$

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Homework #4 Review

2.3 **In Exercises 25–36, solve by completing the square.**

Pg 122 (113) #28. $9x^2 + 2x = 2$

$$x^2 + \frac{1}{3}x = \frac{2}{9}$$
$$\left[\frac{1}{2}\left(\frac{1}{3}\right)\right]^2 = \frac{1}{36}$$
$$x^2 + \frac{1}{3}x + \frac{1}{36} = \frac{2}{9} + \frac{1}{36}$$
$$\left(x + \frac{1}{6}\right)^2 = \frac{1}{4}$$
$$x + \frac{1}{6} = \pm\frac{1}{2}$$
$$x = -\frac{1}{6} \pm \frac{1}{2}$$
$$x = \frac{1}{3} \text{ or } x = -\frac{2}{3}$$

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Homework #4 Review

2.3 **In Exercises 25–36, solve by the quadratic formula.**

Pg 122 (113) #37. $2x^2 + 3x = 0$

$$a = +2 \; b = +3 \; c = 0$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-3 \pm \sqrt{3^2 - 4(2)(0)}}{2(2)}$$
$$= \frac{-3 \pm \sqrt{9}}{4}$$
$$= \frac{-3 \pm 3}{4}$$
$$x = 0 \text{ or } x = -\frac{3}{2}$$

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Homework #4 Review

#40. **Pg 123 (114) "solve by the quadratic formula"**

$$2x^2 - 3x - 2 = 0$$

Thus: $a = +2 \quad b = -3 \quad c = -2$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(-2)}}{2(2)}$$
$$= \frac{3 \pm \sqrt{25}}{4}$$
$$= \frac{3 \pm 5}{4}$$

Therefore, $x = 2$ **and** $x = -\frac{1}{2}$ ***final answer***

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Homework #4 Review

#50. **Pg 123 (114) "solve by any method"**

$$2t^2 + 2t + 3 = 0$$

use the quadratic equation: $a = +2 \quad b = +2 \quad c = +3$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-2 \pm \sqrt{2^2 - 4(2)(3)}}{2(2)}$$
$$= \frac{-2 \pm \sqrt{-20}}{4}$$
$$= \frac{-2 \pm 2\sqrt{5}i}{4}$$
$$= \frac{-1 \pm \sqrt{5}i}{2}$$

Homework #4 Review

#52. **Pg 123 (114) "solve by any method"**

$$x^2 + 2x = 0$$

This is a 'special case' factoring:

$$x\ (x + 2) = 0$$

this becomes $x + 0 = 0;\ x = 0$

and $x + 2 = 0;\ x = -2$

Thus $x = 0$ **or** $x = -2$ ans.

Questions?

Chapter 3.4

Linear Functions REVIEW

The Slope of a Line

The polynomial $f(x) = ax + b$ is a linear function and when graphed, will show a straight line…

We can readily show that if this line is not vertical or horizontal, moving from one point on the line to another can be calculated using the differences in the x's and y's in a ratio called the slope of the line "m" where:

$$m = (y - y_1) / (x - x_1)$$

When **m>0,** *the line graph is an* ***increasing function***

When **m<0,** *the line graph is a* ***decreasing function***

When **m=0,** *the line graph is a* ***constant function***

When ***the line is vertical 'm' does not exist = no function!***

The Point - Slope Formula

We can plot a straight line using the relationship between any two points on that line:

$$m = (y - y_1) / (x - x_1)$$

We can rewrite this as:

$$(y - y_1) = m\ (x - x_1)$$

This is the Point-Slope formula that defines an equation of a line with slope m that passes through the point (x_1, y_1)

The Slope - Intercept Formula

We can also plot a straight line using the more common relationship between any two points on that line:

$$y = m\,x + b$$

This formula defines an equation of a line with slope m and y-intercept b

Note that these formulas do not define horizontal lines, when m = 0:

$$y = b$$

nor vertical lines where a = the x-intercept:

$$x = a$$

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Questions?

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From Words to Algebra

1. Read the problem through the first time to get a general idea of what is being asked.
2. Read the problem a second time to recognize what may be important in determining that which is to be found.
3. If possible, estimate the solution to this problem, and then compare this estimate with your final answer.
4. Let some algebraic symbol denote the quantity to be found.
5. If possible, represent other quantities in the problem in terms of the algebraic symbol designated in Step 4.
6. Find various relationships (equations or inequalities) in the problem.
7. Use relationships established in Step 6 to find the solution to the problem.
8. Verify that your answer is, indeed, the solution to the problem.

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Questions?

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In Chapters 4 & 5

4. Polynomial Functions

- **4.1 Quadratic Functions and Their Graphs**
- **4.2 Graphs of Polynomial Functions of Higher Degree**
- **4.3 Polynomial Division and Synthetic Division**
- **4.4 The Remainder and Factor Theorems**
- **4.5 Factors and Zeros**
- **4.6 Real, Complex and Rational Zeros**
- **4.7 Approximation of the Zeros of a Polynomial Function**

5. Rational Functions and Conic Sections

- **5.1 Rational Functions and their Graphs**
- **5.2 The Circle**
- **5.3 The Parabola**
- **5.4 The Ellipse and Hyperbola**
- **5.5 Translation of Axis**

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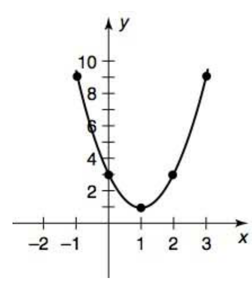
Graphing Quadratic Equations

$$y = f(x) = ax^2 + bx + c \quad \text{where } a \neq 0$$

$$\text{Let } y = f(x) = 2x^2 - 4x + 3$$

For $x = -1$: $y = 2(+1) - 4(-1) + 3 = 2+4+3 = 9$

x	y
−1	9
0	3
1	1
2	3
3	9



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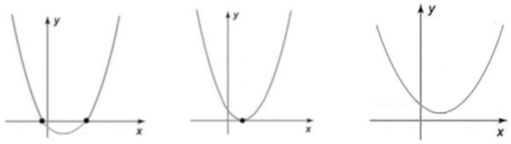
About Roots

For $y = f(x) = ax^2 + bx + c$

y is a Parabola and

there are three possible solutions:

Two Roots ***One Double Root*** ***No Real Roots***



About Chapter 4 & 5

We may discus additional topics from Chapters 4 and 5 later this semester…

Questions?

In Chapter 6

6. Exponential and Logarithmic Functions

- **6.1 A Brief Review of Inverse Functions *(not reviewed)***
- **6.2 Exponential Functions**
- **6.3 Logarithmic Functions**
- **6.4 Fundamental Properties of Logarithms**
- **6.5 Exponential and Logarithmic Equations**

Chapter 6.2

Exponential Functions and their Graphs

Exponential Functions

Up to this point in the course we have dealt mainly with

algebraic functions

These include polynomial functions and rational functions. In this chapter we will study two types of ***non-algebraic*** functions: ***exponential functions*** and ***logarithmic functions.***

These functions are examples of

transcendental functions

Definition of Exponential Function

The **exponential function *f* with base *a*** is denoted by

$f(x) = a^x$

where $a > 0$, $a \neq 1$, and x is any real number.

Exponential Functions

Note that in the definition of an exponential function, the base $a = 1$ is ***excluded*** because it yields

$$f(x) = 1^x = 1$$

The equation with a=1 is a **constant function**, and **not** an *exponential function*.

Exponential Functions

Try these exponential functions

$$f(x) = a^x$$

on your calculator:

Function	*Value*
a. $f(x) = 2^x$	$x = -3.1$

Function Value	Graphing Calculator Keystrokes	Display
a. $f(-3.1) = 2^{-3.1}$	2 ^ (−) 3.1 ENTER	0.1166291

Function	*Value*
b. $f(x) = 2^{-x}$	$x = \pi$

Function Value	Graphing Calculator Keystrokes	Display
b. $f(\pi) = 2^{-\pi}$	2 ^ (−) π ENTER	0.1133147

Exponential Functions

Try these exponential functions

$$f(x) = a^x$$

on your calculator:

Function	*Value*
c. $f(x) = 0.6^x$	$x = \frac{3}{2}$

Function Value	Graphing Calculator Keystrokes	Display
c. $f(\frac{3}{2}) = (0.6)^{3/2}$	.6 ^ (3 ÷ 2) ENTER	0.4647580

Function	*Value*
d. $f(x) = 1.05^{2x}$	$x = 12$

Function Value	Graphing Calculator Keystrokes	Display
d. $f(12) = (1.05)^{2(12)}$	1.05 ^ (2 × 12) ENTER	3.2250999

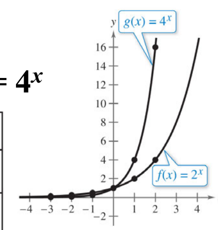
Graphs of Exponential Functions

Graphs of all exponential functions have similar characteristics…

Graphs of $y = a^x$

Let $f(x) = 2^x$ and $g(x) = 4^x$

x	-3	-2	-1	0	1	2
2^x	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4
4^x	$\frac{1}{64}$	$\frac{1}{16}$	$\frac{1}{4}$	1	4	16



Plotting these points on the same coordinates and connecting them with smooth curves shows the graphs of exponential functions approach infinity

Graphs of Exponential Functions

The *parent exponential function*

$$f(x) = a^x,\ a > 0,\ a \neq 1$$

is different from all the functions studied so far because the variable x is an *exponent*.

A distinguishing characteristic of an exponential function is its rapid increase as x increases (for $a > 1$).

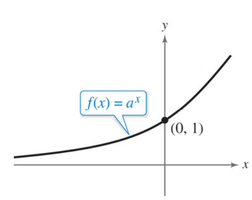
Graph of $f(x) = a^x$, $a > 1$	*Graph of* $f(x) = a^{-x}$, $a > 1$
Domain: $(-\infty, \infty)$	Domain: $(-\infty, \infty)$
Range: $(0, \infty)$	Range: $(0, \infty)$
Intercept: $(0, 1)$	Intercept: $(0, 1)$
Increasing on: $(-\infty, \infty)$	Decreasing on: $(-\infty, \infty)$

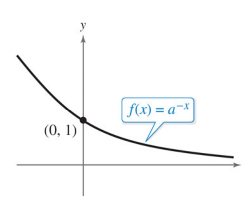
Graphs of Exponential Functions

Exponential Graphs have

x-axis as a horizontal asymptote

($a^x \to 0$ as $x \to -\infty$) ($a^{-x} \to 0$ as $x \to \infty$)





The Natural Base e

For many applications, the convenient choice for a base is the irrational number

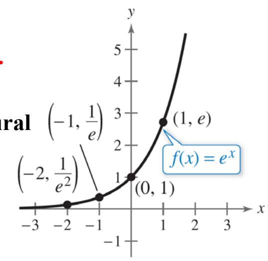
$e = 2.718281828\ldots$

This number is called the **natural base**.

The function

$f(x) = e^x$

is called the **natural exponential function**



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Natural Exponential Functions

Use a calculator to evaluate the function

$f(x) = e^x$

at each indicated value of x.

a. $x = -2$

Function Value	Graphing Calculator Keystrokes	Display
a. $f(-2) = e^{-2}$	e^x (−) 2 ENTER	0.1353353

b. $x = 0.25$

Function Value	Graphing Calculator Keystrokes	Display
b. $f(0.25) = e^{0.25}$	e^x .25 ENTER	1.2840254

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Natural Exponential Functions

Use a calculator to evaluate the function

$f(x) = e^x$

at each indicated value of x.

c. $x = -0.4$

Function Value	Graphing Calculator Keystrokes	Display
c. $f(-0.4) = e^{-0.4}$	e^x (−) .4 ENTER	0.6703200

d. $x = \frac{2}{3}$

Function Value	Graphing Calculator Keystrokes	Display
d. $f\left(\frac{2}{3}\right) = e^{2/3}$	e^x (2 ÷ 3) ENTER	1.9477340

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Exponential Function Applications

One of the most familiar examples of exponential growth is an investment earning ***continuously compounded interest***.

Suppose a principal P is invested at an annual interest rate r, compounded ***once a year***.

If the interest is added to the principal at the end of the year, then the new balance P_1 is

$$P_1 = P + Pr = P(1 + r)$$

This pattern of multiplying the previous principal by $1 + r$ is then repeated each successive year, as shown in the table:

Time in years	Balance after each compounding
0	$P = P$
1	$P_1 = P(1 + r)$
2	$P_2 = P_1(1 + r) = P(1 + r)(1 + r) = P(1 + r)^2$
⋮	⋮
t	$P_t = P(1 + r)^t$

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About Compound Interest…

Then the interest rate per compounding period is r/n, and the account balance after t years is

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

Amount (*balance*) with n compoundings per year

When the number of compoundings n increases without bound, the process approaches what is called **continuous compounding**. In the formula for n compoundings per year, let $m = n/r$. This produces

$$A = P\left(1 + \frac{r}{n}\right)^{nt} = P\left(1 + \frac{1}{m}\right)^{mrt} = P\left[\left(1 + \frac{1}{m}\right)^m\right]^{rt}$$

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Exponential Function Applications

As m increases without bound, we see that

$\left(1 + \frac{1}{m}\right)^m$ approaches e.

So, for *continuous compounding*, it follows that

$$P\left[\left(1 + \frac{1}{m}\right)^m\right]^{rt} \Rightarrow P[e]^{rt}$$

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Compound Interest

Formulas for Compound Interest

After t years, the balance A in an account with principal P and annual interest rate r (in decimal form) is given by the following formulas:

1. For n compoundings per year: $A = P\left(1 + \frac{r}{n}\right)^{nt}$
2. For continuous compounding: $A = Pe^{rt}$

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Interest Rate Example …

A total of \$9000 is invested at an annual interest rate of 2.5%, compounded annually. Find the balance in the account after 5 years using the equation:

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

You have five minutes to calculate…

ANS: for $P = 9000$, $r = 2.5\% = 0.025$, $n = 1$, and $t = 5$

Substituting:

$$A = 9000\left(1 + \frac{0.025}{1}\right)^{(1)(5)}$$

Simplify: $A = 9000\,(1.025)^{5}$

Calculate: $A \approx \$10{,}182.67$

The balance in the account after 5 years is **\$10,182.67**

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Chapter 6.3

Logarithmic Functions and their Graphs

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Logarithmic Functions

When a function is one-to-one—that is, when the function has the property that no horizontal line intersects its graph more than once—the function must have an inverse function.

Every function of the form

$$f(x) = a^x, \text{ where } a > 0 \text{ and } a \neq 1$$

passes the Horizontal Line Test and therefore must have an inverse function. This inverse function is called the **logarithmic function with base *a*.**

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Logarithmic Functions

Definition of a Logarithmic Function

For $x > 0$, $a > 0$ and $a \neq 1$ …

$f(x) = y = \log_a x$ *if and only if* $x = a^y$

The function given by

$f(x) = \log_a x$ *(read as "log base a of x")*

is called the ***logarithmic function with base a***.

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Logarithmic Functions

Every logarithmic equation can be written in an equivalent exponential form and

Every exponential equation can be written in logarithmic form.

So, the equations

$y = \log_a x$ and $x = a^y$

are equivalent.

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Evaluating Logarithms

Evaluate each logarithm at the indicated value of x.

You have three minutes per question.

a. $f(x) = \log_2 x$, $x = 32$

$f(32) = \log_2 32 = 5$ *because* $2^5 = 32$

b. $f(x) = \log_3 x$, $x = 1$

$f(1) = \log_3 1 = 0$ *because* $3^0 = 1$

Evaluating Logarithms

Evaluate each logarithm at the indicated value of x.

You have three minutes per question.

c. $f(x) = \log_4 x$, $x = 2$

$f(2) = \log_4 2 = \frac{1}{2}$ *because* $4^{1/2} = \sqrt{4} = 2$

d. $f(x) = \log_{10} x$, $x = \frac{1}{100}$

$f(\frac{1}{100}) = \log_{10} \frac{1}{100} = -2$ *because* $10^{-2} = \frac{1}{10^2} = \frac{1}{100}$

Common Logarithmic Functions

The logarithmic function with base 10 is called the **common logarithmic function**

Properties of Logarithms:

1. $\log_a 1 = 0$ *because* $a^0 = 1$
2. $\log_a a = 1$ *because* $a^1 = a$
3. $\log_a a^x = x$ *and* $a^{\log_a x} = x$
   This is the Inverse Property
4. If $\log_a x = \log_a y$ *then* $x = y$
   This is the One-to-One Property

Logarithmic Functions

Solve these equations for x:

You have three minutes per question.

a. $\log_2 x = \log_2 3$

Using the One-to-One Property (Property 4),

[If $\log_a x = \log_a y$ *then* $x = y$]

you can conclude that $x = 3$

b. $\log_4 4 = x$

Using Property 2,

[$\log_a a = 1$ *because* $a^1 = a$]

you can conclude that $x = 1$

Logarithmic Functions

Simplify:

You have three minutes per question.

c. $\log_5 5^x$

Using the Inverse Property (Property 3),

[$\log_a a^x = x$ *and* $a^{\log_a x} = x$]

it follows that

$\log_5 5^x = x$

d. $7^{\log_7 14}$

Using the Inverse Property (Property 3), *(above)*

it follows that

$7^{\log_7 14} = 14$

Graphs of Logarithmic Functions

To sketch the graph of $y = \log_a x$, you can use the fact that the graphs of inverse functions are reflections of each other in the line $y = x$.

In the same coordinate plane, sketch the graph of each function by hand:

a. $f(x) = 2^x$

b. $g(x) = \log_2 x$

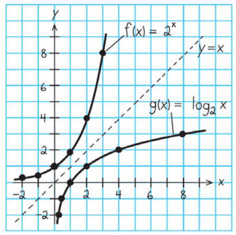
For $f(x) = 2^x$, construct a table of values

x	-2	-1	0	1	2	3
$f(x) = 2^x$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8

Graphs of Logarithmic Functions

By plotting the points on the "a" table and connecting them with a smooth curve, you obtain the graph of ***f*** :

b. Because $g(x) = \log_2 x$ is the ***inverse function*** of $f(x) = 2^x$, the graph of ***g*** is obtained by plotting the points $(f(x), x)$ and connecting them with a smooth curve.



The graph of ***g*** is a reflection of the graph of ***f*** 'around' the line $y = x$

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Parent Logarithmic Functions

The ***parent logarithmic function***

$$f(x) = \log_a x,\ a > 0,\ a \neq 1$$

is the inverse function of the exponential function. Its domain is the set of positive real numbers and its range is the set of all real numbers. This is the opposite of the exponential function.

Moreover, the **logarithmic function** has

the ***y-axis as a vertical asymptote***,

whereas the **exponential function** has

the ***x-axis as a horizontal asymptote***

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Logarithmic Functions

Many real-life phenomena with slow rates of growth can be modeled by logarithmic functions. The basic characteristics of the logarithmic function are summarized below

Graph of $f(x) = \log_a x,\ a > 1$

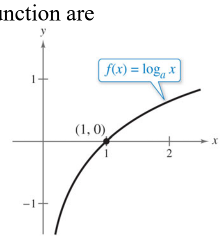
Domain: $(0, \infty)$

Range: $(-\infty, \infty)$

Intercept: (1, 0)

Increasing on: $(0, \infty)$

y-axis is a vertical asymptote ($\log_a x \to -\infty$ as $x \to 0^+$)



The graph of $f(x) = a^x$ is a ***Continuous Reflection*** of the line $y = x$

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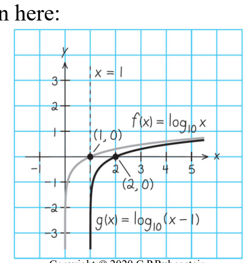
Parent function $f(x) = \log_a x$

Each of the following functions is a ***transformation*** of the graph of $f(x) = \log_{10} x$

a. Because $g(x) = \log_{10}(x - 1) = f(x - 1)$, the graph of ***g*** can be obtained by shifting the graph of ***f*** one unit to the *right*, as shown here:



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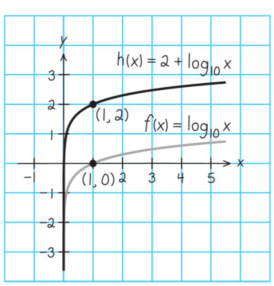
Parent function: $f(x) = \log_a x$

b. Because $h(x) = 2 + \log_{10} x = 2 + f(x)$, the graph of ***h*** can be obtained by shifting the graph of ***f*** two units *upward*:



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Natural Logarithmic Functions

The Natural Exponential Function $f(x) = e^x$ is one-to-one and so it has an inverse function.

That inverse function is called

The **natural logarithmic function**

and is denoted by the special symbol **ln *x***, read as "**the natural log of *x***" or "**el en of *x***"

For $x > 0$, $y = \ln x$ *if and only if* $x = e^y$

The function given by

$$f(x) = \log_e x = \ln x$$

is called the **natural logarithmic function**

The equations $y = \ln x$ and $x = e^y$ are equivalent.

Note that the natural logarithm **ln *x*** *is written without a base - the base is understood to be* ***e***.

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Natural Logarithms

Use a calculator to evaluate the function $f(x) = \ln x$ at each value of x.

You have three minutes per question.

a. $x = 2$

Function Value	Graphing Calculator Keystrokes	Display
a. $f(2) = \ln 2$	LN 2 ENTER	0.6931472

b. $x = 0.3$

Function Value	Graphing Calculator Keystrokes	Display
b. $f(0.3) = \ln 0.3$	LN .3 ENTER	–1.2039728

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Natural Logarithms

Use a calculator to evaluate the function $f(x) = \ln x$ at each value of x.

You have three minutes per question.

c. $x = -1$

Function Value	Graphing Calculator Keystrokes	Display
c. $f(-1) = \ln(-1)$	LN (–) 1 ENTER	ERROR

d. $x = 1 + \sqrt{2}$

Function Value	Graphing Calculator Keystrokes	Display
d. $f(1+\sqrt{2}) = \ln(1+\sqrt{2})$	LN (1 + √ 2) ENTER	0.8813736

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Natural Logarithm Properties

The *following* properties of logarithms are valid for natural logarithms:

Properties of Natural Logarithms

1. $\ln 1 = 0$ *because* $e^0 = 1$
2. $\ln e = 1$ *because* $e^1 = e$
3. $\ln e^x = x$ *and* $e^{\ln x} = x$
   This is the Inverse Property
4. If $\ln x = \ln y$ *then* $x = y$
   This is the One-to-One Property

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Logarithmic Functions

Use the properties of natural logarithms to rewrite each expression.

You have three minutes per question.

c. 4 ln 1

Solution

c. $4 \ln 1 = 4(0) = 0$

Property 1 $\ln 1 = 0$ *because* $e^0 = 1$

d. 2 ln *e*

Solution

d. $2 \ln e = 2(1) = 2$

Property 2 $\ln e = 1$ *because* $e^1 = e$

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Domains of Logarithmic Functions

Find the domain of each function.

a. $f(x) = \ln(x - 2)$

Solution:

a. Because $\ln(x - 2)$ is defined only when

$$x - 2 > 0$$

it follows that the domain of f is $(2, \infty)$.

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Logarithmic Functions

Find the domain of each function.

b. $g(x) = \ln(2 - x)$

b. Because $\ln(2 - x)$ is defined only when

$$2 - x > 0$$

it follows that the domain of g is $(-\infty, 2)$.

c. $h(x) = \ln x^2$

c. Because $\ln x^2$ is defined only when

$$x^2 > 0$$

it follows that the domain of h is all real numbers except $x = 0$

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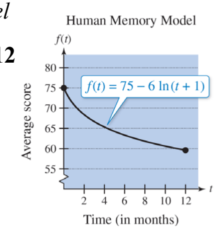
Applications of Logarithmic Functions

Students participating in a psychology experiment attended several lectures on a subject and were given an exam. Every month for a year after the exam, the students were retested to see how much of the material they remembered. The average scores for the group are given by the *human memory model*

$$f(t) = 75 - 6\ln(t + 1), \quad 0 \le t \le 12$$

where t is the time in months.
The graph of f is seen here →



Logarithmic Function Example

$$f(t) = 75 - 6\ln(t + 1), \quad 0 \le t \le 12$$

a. What was the average score on the original *human memory model* exam ($t = 0$)?

You have three minutes per question.

Solution:

a. The original average score was

$f(0) = 75 - 6\ln(0 + 1)$

$= 75 - 6\ln 1$

$= 75 - 6(0)$

$= 75$

Logarithmic Function Example

$$f(t) = 75 - 6\ln(t + 1), \quad 0 \le t \le 12$$

b. What was the average score on the *human memory model* at the end of $t = 2$ months?

You have three minutes per question.

b. After 2 months, the average score was

$f(2) = 75 - 6\ln(2 + 1)$

$= 75 - 6\ln 3$

$\approx 75 - 6(1.0986)$

≈ 68.41

Logarithmic Function Example

$$f(t) = 75 - 6\ln(t + 1), \quad 0 \le t \le 12$$

c. What was the average score on the *human memory model* at the end of $t = 6$ months?

You have three minutes per question.

c. After 6 months, the average score was

$f(6) = 75 - 6\ln(6 + 1)$

$= 75 - 6\ln 7$

$\approx 75 - 6(1.9459)$

≈ 63.32

Next Class Session #6

Class Sessions Posted Online Friday before Class

- ***DUE:*** ***Homework Set #05 by 12:00Noon 12 October!***
- ***NOTE: Quiz 5 = four problems from hwk***
- ***Review:*** ***Homework Set #05; Textbook readings***
- ***Lecture: Exponential and Logarithmic Functions, continued***

**** Exam #1 to be Emailed to all immediately after class ****

Exam 1 MUST BE RETURNED VIA EMAIL by 11:00pm

<u>*In class* – Session 7: Monday 19 October:</u>

- ***DUE:*** ***Homework Set #06 by 12:00Noon 19 October!***
  NOTE: Quiz 6 = four problems from hwk
- ***Review: Exam #1,*** ***Homework Set #06; Textbook readings***
- ***Lecture: Functions, Graphing, Exponents and Logarithms***

Any Questions?
Send me an email …

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End

