Pratt

Math 150 – Fall 2020
Algebra & Trigonometry

Charles Rubenstein, Ph. D.
Professor of Engineering & Information Science

Session 6: Monday 10/12/20

6:30pm - 9:20pm

via REMOTE LEARNING

Revision 1

Instructor Contact Information

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Professor of Engineering & Information Science

Faculty Office: ARC G-49

Fall 2020 Virtual Office hours *ONLY*

Wednesdays 10:00am-2:00pm via Zoom Meeting

To make your appointment

Send me an email at least one day in advance :

crubenst@pratt.edu

or ***c.rubenstein@ieee.org***

This Class Session #06

Class Sessions Posted Online Friday before Class

***In class* – Session 6: Monday 12 October:**

- ***DUE:* Homework Set #05 by 12:00Noon 12 October!**
- ***NOTE: Quiz 5 = four problems from hwk (*)***
- ***Review:* Textbook readings**
- ***Lecture: Exponential and Logarithmic Functions, continued***

Exam #1 to be Emailed to all immediately after class

MUST BE RETURNED VIA EMAIL by 11:00pm

***In class* – Session 7: Monday 19 October:**

- ***DUE:* Homework Set #06 by 12:00Noon 12 October!**
- ***NOTE: Quiz 6 = four problems from hwk***
- ***Review: Exam 1; Homework Set #05; Textbook readings***
- ***Lecture: Functions, continued***

(* NOTE: I have been emailing the answers to the homework BEFORE the due date in error…)

About the Homework Quizzes

I have selected four (4) problems from each homework for you to submit and - as long as at least three are answered correctly - receive 'quiz' credit of 3% for correct answers.

These are the selected problems for the remaining homework assignments:

HWK #06: Section 2.2: 54a, 54c, 55, 57

HWK #07: Section 3.1: 10, 60 and Section 3.4: 10, 16
HWK #08: Section 1.4: 82c and Section 1.5: 42, 54, 60
HWL #09: Number: 2, 4, 6, 8
HWK #10: Section 7.8: 4, 8, 10, 14

Homework is due not later than 12:00pm Noon ET on day of our class session.

If not emailed by then, a zero grade will be entered

Emailing me your Homework

As noted, I have selected four (4) problems from each homework for you to submit each week per the previous slide.

Homework is due not later than Noon class days.

HOW TO PREPARE YOUR ASSIGNMENT:

1. Use DARK BLACK pencil or pen on White Paper.

If I can't read your work you get a ZERO!

2. Please scan your work as a PDF and save it as ***lastname_xx.pdf***

HOWEVER – IF YOU CAN NOT SCAN –

a. Take a photo of your work

b. Insert the photo(s) into a Word document

c. Save as ***lastname_xx.docx*** *or* ***lastname_xx.pdf***

Then email your file to me: ***crubenst@pratt.edu***

Email me ONLY the requested four (4) problems.

(Email any you might be challenged by in a separate document)

About Exam #1 – Worth 20%

Exam #1 is a one hour exam with 7 @ 10 point and 6 @ 5 point questions that will be emailed after our Zoom Class on Monday 12 October (by 8:00pmET)

***EXAM 1 is DUE* by 11:00pm 12 October!**

HOW TO EMAIL ME YOUR EXAM:

1. You MUST use DARK BLACK pencil or pen on white paper.

If I can't read your work you get a ZERO!

2. Please scan your work as a PDF and save it as ***lastname_E1.pdf***

HOWEVER – IF YOU CAN NOT SCAN –

Fill out the docx file. Take a photo of any work unable to be 'typed out' and insert the photo(s) into the space allotted and save the file as: ***lastname_E1.docx*** *and attach the file (**NO CLOUD LINKS**)*

Include the worked out problems AND solutions AND any units…

Email your file to me at: ***crubenst@pratt.edu***

With the Subject Line: ***Math150 Exam***

Math 150 – Class Topics

1. The Foundations of Algebra
2. Equations and Inequalities
3. Functions
4. Polynomial Functions
5. Rational Functions and Conic Sections
6. Exponential and Logarithmic Functions
7. The Trigonometric Functions
8. Analytic Trigonometry
9. Applications of Trigonometry
10. Systems of Equations and Inequalities
11. Matrices, Linear Systems, and Determinants
12. Topics in Algebra

Draft Schedule: Math 150 – Fall 2020 – Remote Learning

Monday	Notes
24-Aug	1. Introduction: Numbers, Arithmetic Operations, Fractions
31-Aug	2. Manipulation of Algebraic Expressions; *Hwk #1 Due @ Noon*
7-Sep	NO CLASSES – Labor Day
14-Sep	3. Solving Linear and Quadratic Equations of One Variable; *Hwk #2 Due*
21-Sep	4. Solving Equations of Two Variables; *Hwk #3 Due*
28-Sep	NO CLASSES – Instructor Holiday
5-Oct	5. Creating Equations: Polynomials, Exponents & Logarithms *Hwk #4 Due*
12-Oct	6. Functions, Graphing, Exponents and Logarithms; *Hwk #5; Exam #1*
19-Oct	7. Exponents and Logarithms, Continued; *Hwk #6; Exam Review*
26-Oct	8. Trigonometric Functions, Pythagorean Theorem; *Hwk #7 Due*
2-Nov	9. Applications of Trigonometry; *Hwk #8 Due*
9-Nov	10. Analytic Trigonometry: Identities & Graphing; *Hwk #9 Due; Exam #2*
16-Nov	11. Areas and Volumes of Geometric Solids; *Hwk #10; Exam Review*
23-Nov	12. Systems of Equations and Inequalities
30-Nov	13. Series and Sequences, Review topics
7-Dec	Final Examination *Exam Emailed Monday 9:00am - Due at 1:00pm ET ?*

www.CharlesRubenstein.com/150

20fa06.pdf = *This slide set**

20fa06_h.pdf = *slides as 6-up handouts**

**My goal is to post these not later than Noon on the Friday one week before our Zoom Class Meetings*

Questions?

Homework #05
Selected Problems
will be Reviewed
NEXT WEEK

In Chapter 6

6. Exponential and Logarithmic Functions

- 6.1 A Brief Review of Inverse Functions *(not reviewed)*
- 6.2 Exponential Functions
- 6.3 Logarithmic Functions
- 6.4 Fundamental Properties of Logarithms
- 6.5 Exponential and Logarithmic Equations

Chapter 6.4

Fundamental Properties of Logarithms

Logarithms: Change of Base

Most calculators have only two types of log keys, one for common logarithms (***base 10***) and one for natural logarithms (**base *e***). Although common logs and natural logs are the most frequently used, you may occasionally need to evaluate logarithms to other bases using the **change-of-base formula:**

Let ***a***, ***b***, and ***x*** be positive real numbers such that $a \neq 1$ and $b \neq 1$

Then $\log_a x$ can be converted to a different base using these:

Base ***b***

$$\log_a x = \frac{\log_b x}{\log_b a}$$

Base ***10***

$$\log_a x = \frac{\log_{10} x}{\log_{10} a}$$

Base ***e***

$$\log_a x = \frac{\ln x}{\ln a}$$

One way to look at the change-of-base formula is that *logarithms to base **a** are simply constant multiples of logarithms to base **b***.

The constant multiplier is $\dfrac{1}{\log_b a}$

Changing Bases Using Common Logarithms

$$\log_a x = \frac{\log_{10} x}{\log_{10} a}$$

a. $\text{Log}_4\ 25 = \dfrac{\log_{10} 25}{\log_{10} 4}$ *Now use your Calculator*

$\approx \dfrac{1.39794}{0.60206}$ *and Simplify*

≈ 2.32

b. $\text{Log}_3\ 17 = \dfrac{\log_{10} 17}{\log_{10} 3}$

$\approx \dfrac{1.23045}{0.47712}$

≈ 3.58

Properties of Logarithms

Let a be a positive real number such that $a \neq 1$ and let n be a real number. If u and v are positive real numbers, then the following properties are true:

	Logarithm with Base a	*Natural Logarithm*
1. Product Property:	$\log_a(uv) = \log_a u + \log_a v$	$\ln(uv) = \ln u + \ln v$
2. Quotient Property:	$\log_a \frac{u}{v} = \log_a u - \log_a v$	$\ln \frac{u}{v} = \ln u - \ln v$
3. Power Property:	$\log_a u^n = n \log_a u$	$\ln u^n = n \ln u$

Using Properties of Logarithms

Write each logarithm in terms of **ln 2** and **ln 3**.

a. ln 6

Solution:

a. $\ln 6 = \ln(2 \cdot 3)$ Rewrite 6 as $2 \cdot 3$.

$= \ln 2 + \ln 3$ Product Property

b. $\ln \frac{2}{27}$

Solution:

b. $\ln \frac{2}{27} = \ln 2 - \ln 27$ Quotient Property

$= \ln 2 - \ln 3^3$ Rewrite 27 as 3^3.

$= \ln 2 - 3 \ln 3$ Power Property

Rewriting Logarithmic Expressions

The properties of logarithms are useful for rewriting logarithmic expressions in forms that simplify the operations of algebra.

This is true because they convert complicated products, quotients, and exponential forms

into simpler sums, differences, and products, respectively.

Expanding Logarithmic Expressions

Using the properties of logarithms to expand expressions.

a. $\log_4 5x^3y$

Solution:

a. $\log_4 5x^3y = \log_4 5 + \log_4 x^3 + \log_4 y$ Product Property

$= \log_4 5 + 3\log_4 x + \log_4 y$ Power Property

b. $\ln \dfrac{\sqrt{3x-5}}{7}$

Solution:

b. $\ln \dfrac{\sqrt{3x-5}}{7} = \ln \dfrac{(3x-5)^{1/2}}{7}$ Rewrite radical using a rational exponent.

$= \ln(3x-5)^{1/2} - \ln 7$ Quotient Property

$= \dfrac{1}{2}\ln(3x-5) - \ln 7$ Power Property

Condensing Logarithmic Expressions

Use the properties of logarithms to condense each expression.

a. $\frac{1}{2}\log_{10} x + 3\log_{10}(x+1)$

Solution:

a. $\frac{1}{2}\log_{10} x + 3\log_{10}(x+1)$

$= \log_{10} x^{1/2} + \log_{10}(x+1)^3$ Power Property

$= \log_{10}\left[\sqrt{x}(x+1)^3\right]$ Product Property

b. $2\ln(x+2) - \ln x$

Solution:

b. $2\ln(x+2) - \ln x = \ln(x+2)^2 - \ln x$ Power Property

$= \ln\dfrac{(x+2)^2}{x}$ Quotient Property

Finding a Mathematical Model

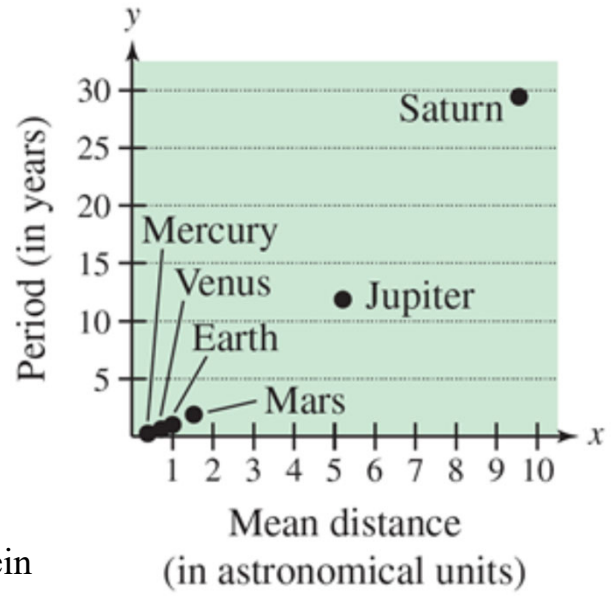
This table shows the mean distance ***x*** from the sun and the period ***y*** (the time it takes a planet to orbit the sun) for each of the six planets that are closest to the sun.

In the table, the mean distance is given in ***astronomical units*** (where the Earth's mean distance is defined as 1.0), and the period is given in ***years***.

DATA

Planet	Mercury	Venus	Earth	Mars	Jupiter	Saturn
Mean distance, *x*	0.387	0.723	1.000	1.524	5.203	9.537
Period, *y*	0.241	0.615	1.000	1.881	11.863	29.447

The points in the table are plotted →



And our task is to:

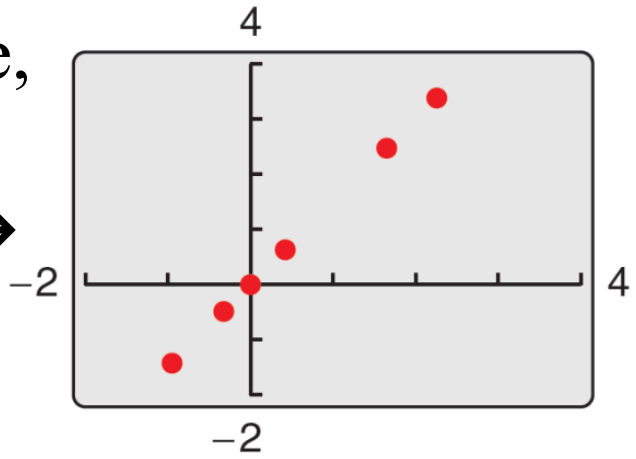
Find an equation that relates y and x.

Solution…

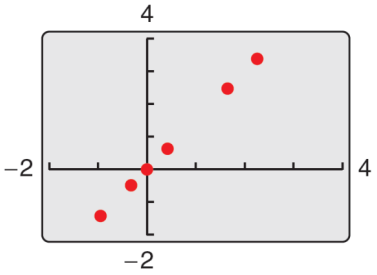
From the figure given, it is not clear how to find an equation that relates y and x. To solve this problem, take the **natural log** of each of the ***x*-values** and ***y*-values** in the table:

Planet	Mercury	Venus	Earth	Mars	Jupiter	Saturn
$\ln x = X$	−0.949	−0.324	0.000	0.421	1.649	2.255
$\ln y = Y$	−1.423	−0.486	0.000	0.632	2.473	3.383

Now, by plotting the points in the table, you can see that all six of the points appear to lie in a line, as shown here →



Logarithms



To find an equation of the line through these points, you can use algebraic methods: Choose any *two points* to determine the ***slope*** of the line. Using the two points **(0.421, 0.632)** and **(0, 0)**, you can determine that the slope of the line is

$$m = \frac{0.632 - 0}{0.421 - 0}$$ thus **m ≈ 1.5** or **m = 3/2**

By the ***point-slope formula***, the equation of the line is

$$Y = \tfrac{3}{2}X$$

Where ***Y* = ln *y*** and ***X* = ln *x***

you can therefore conclude that

$$\ln y = \frac{3}{2} \ln x.$$

Chapter 6.5

Exponential and Logarithmic Equations

Exponential & Logarithm Problems

There are two basic strategies for solving exponential or logarithmic equations. The first is based on the One-to-One Properties and the second is based on the Inverse Properties.

For $a > 0$ and $a \neq 1$, the following properties are true for all x and y for which $\log_a x$ and $\log_a y$ are defined.

One-to-One Properties:

$$a^x = a^y \quad \textit{if and only if} \quad x = y.$$

$$\log_a x = \log_a y \quad \textit{if and only if} \quad x = y.$$

Inverse Properties:

$$a^{\log_a x} = x$$

$$\log_a a^x = x$$

One-to-One Logarithm Problems

Using the One-to-One Property Rule

Original Equation	*Rewritten Equation*	*Solution*
a. $2^x = 32$	$2^x = 2^5$	$x = 5$
b. $\log_4 x - \log_4 8 = 0$	$\log_4 x = \log_4 8$	$x = 8$
c. $\ln x - \ln 3 = 0$	$\ln x = \ln 3$	$x = 3$
d. $\left(\frac{1}{3}\right)^x = 9$	$3^{-x} = 3^2$	$x = -2$

Inverse Property Logarithm Problems

Using the Inverse Property Rule

Original Equation	*Rewritten Equation*	*Solution*
a. $e^x = 7$	$\ln e^x = \ln 7$	$x = \ln 7$
b. $\ln x = -3$	$e^{\ln x} = e^{-3}$	$x = e^{-3}$
c. $\log_{10} x = -1$	$10^{\log_{10} x} = 10^{-1}$	$x = 10^{-1} = \frac{1}{10}$
d. $\log_3 x = 4$	$3^{\log_3 x} = 3^4$	$x = 81$

Strategies

for Solving Exponential and Logarithmic Equations

1. Rewrite the original equation in a form that allows the use of **One-to-One Properties** of exponential and logarithmic functions.

2. Rewrite an ***exponential*** equation in logarithmic form and apply the **Inverse Property of logarithmic functions.**

3. Rewrite a ***logarithmic*** equation in exponential form and apply the **Inverse Property of exponential functions.**

Solving Exponential Equations

Solve and check solutions in the original equation.

a. $e^x = 72$

Solution:

a. $e^x = 72$ Write original equation.

$\ln e^x = \ln 72$ Take natural log of each side.

$x = \ln 72 \approx \boldsymbol{4.28}$ Inverse Property; Use Calculator

b. $3(2^x) = 42$

Solution: $2^x = 14$ After dividing each side by 3.

$\log_2 2^x = \log_2 14$ Take log (base 2) of each side.

$x = \log_2 14$ Inverse Property

$x = \dfrac{\ln 14}{\ln 2} \approx 3.81$ Change-of-base formula; use a Calculator

Solving Logarithm Equations

To solve a logarithmic equation, you can write it in exponential form.

$\ln x = 3$ Logarithmic form

$e^{\ln x} = e^{3}$ Exponentiate each side.

$x = e^{3}$ Exponential form

This procedure is called ***exponentiating*** each side of an equation. It is applied after the logarithmic expression has been isolated.

Solving Logarithm Problems

Solve each logarithmic equation.

a. $\ln 3x = 2$

Solution:

a.	$\ln 3x = 2$	Write original equation
	$e^{\ln 3x} = e^{2}$	Exponentiate each side.
	$3x = e^{2}$	Inverse Property
	$x = \frac{1}{3}e^{2}$	Divide each side by 3
	$x \approx 2.46$	Use a calculator

Solving Logarithm Problems

Solve each logarithmic equation.

b. $\log_3(5x - 1) = \log_3(x + 7)$

Solution:

b. $\log_3(5x - 1) = \log_3(x + 7)$ Write original equation.

$5x - 1 = x + 7$ One-to-One Property

$x = 2$ Use a calculator.

The solution $x = 2$.

Check this in the original equation.

Proving the Change of Base Formula

Because the domain of a logarithmic function generally does not include all real numbers, you should be sure to check for extraneous solutions of logarithmic equations.

Proving the **Change of Base** Formula: $\log_a x = \dfrac{\log_b x}{\log_b a}$

Begin by letting $y = \log_a x$

and writing the equivalent exponential form: $a^y = x$

Now, taking the logarithms *with base* b of each side produces the following.

$$\log_b a^y = \log_b x$$

$$y \log_b a = \log_b x \qquad \text{Power Property}$$

$$y = \frac{\log_b x}{\log_b a} \text{ and thus } \log_a x = \frac{\log_b x}{\log_b a}$$

Divide each side by $\log_b a$ and replace y with $\log_a x$

Applications

Doubling an Investment - 1

You have deposited **$500** in an account that pays **6.75%** interest, compounded continuously.

How long will it take your money to double?

Solution:

Using the formula for continuous compounding, you can find that the balance in the account is

$$A = Pe^{rt}$$

$$= 500\, e^{0.0675\, t}$$

To find the time required for the balance to double, let $A = 1000$ and solve the resulting equation for t…

Applications

Doubling an Investment - 2

$$A = Pe^{rt}$$

$$A = 500e^{0.0675t}$$

To find the time required for the balance to double, let $A = 1000$ and solve the resulting equation for t:

$1000 = 500e^{0.0675t}$ Substitute 1000 for A.

$e^{0.0675t} = 2$ Divide each side by 500.

$\ln e^{0.0675t} = \ln 2$ Take natural log of each side.

$0.0675t = \ln 2$ Inverse Property

$t = \dfrac{\ln 2}{0.0675}$ Divide each side by 0.0675.

$t \approx 10.27$ Use a calculator.

The balance in the account will double after approximately 10.27 years.

Questions?

Next Class Session #7

Class Sessions Posted Online Friday before Class

Exam #1 Emailed after class TODAY, 12 October

In class – Session 7: Monday 19 October:

- ***DUE:*** ***Homework Set #06 by 12:00Noon 19 October!***
- ***NOTE: Quiz 6 = four problems from hwk***
- ***Review: Exam #1, Homework Set #05; Textbook readings***
- ***Lecture: Functions, continued***

In class – Session 8: Monday 26 October:

- ***DUE:*** ***Homework Set #07 by 12:00Noon 26 October!***

  NOTE: Quiz 7 = four problems from hwk
- ***Review:*** ***Homework Set #06; Textbook readings***
- ***Lecture: Trigonometric Functions***

Exam #1

7 @ 10 point Questions

6 @ 5 point Questions

will be emailed to all immediately at the end of class.

It is due back not later than 11:00pm ET tonight!

Any Questions?

Send me an email …

crubenst@pratt.edu

or

c.rubenstein@ieee.org

End