Pratt

Math 150 – Fall 2020

Algebra & Trigonometry

Charles Rubenstein, Ph. D.
Professor of Engineering & Information Science

Session 10: Monday 11/09/20

6:30pm - 9:20pm

via REMOTE LEARNING

Revision 1

Instructor Contact Information

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Professor of Engineering & Information Science
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Fall 2020 Virtual Office hours *ONLY*

Wednesdays 10:00am-2:00pm via Zoom Meeting

To make your appointment

Send me an email at least one day in advance :

crubenst@pratt.edu

or ***c.rubenstein@ieee.org***

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About the Homework Quizzes

I have selected four (4) problems from each homework for you to submit and - as long as at least three are answered correctly - receive 'quiz' credit of 3% for correct answers.

These are the selected problems for the remaining homework assignments:

HWK #10: Section 7.8: 4, 8, 10, 14

Homework is due not later than 12:00pm Noon ET on day of our class session.

If not emailed by then, a zero grade will be entered

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Emailing me your Homework

As noted, I have selected four (4) problems from each homework for you to submit each week per the previous slide.

Homework is due not later than Noon class days.

HOW TO PREPARE YOUR ASSIGNMENT:

1. Use DARK BLACK pencil or pen on WHITE paper.

If I can't read your work you get a ZERO!

2. Please scan your work as a PDF and save it as ***lastname_xx.pdf***

HOWEVER – IF YOU CAN NOT SCAN –

a. *Take a photo of your work*
b. *Insert the photo(s) into a Word document*
c. *Save as* ***lastname_xx.docx*** *or* ***lastname_xx.pdf***

Then email your file to me: ***crubenst@pratt.edu***

Email me ONLY the requested four (4) problems.

(Email any you might be challenged by in a separate document)

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Math 150 – Class Topics

1. **The Foundations of Algebra**
2. **Equations and Inequalities**
3. **Functions**
4. **Polynomial Functions**
5. **Rational Functions and Conic Sections**
6. **Exponential and Logarithmic Functions**
7. **The Trigonometric Functions**
8. **Analytic Trigonometry**
9. **Applications of Trigonometry**
10. **Systems of Equations and Inequalities**
11. **Matrices, Linear Systems, and Determinants**
12. **Topics in Algebra**

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Draft Schedule: Math 150 – Fall 2020 – Remote Learning

Monday	Notes
24-Aug	1. Introduction: Numbers, Arithmetic Operations, Fractions
31-Aug	2. Manipulation of Algebraic Expressions; *Hwk #1 Due @ Noon*
7-Sep	NO CLASSES – Labor Day
14-Sep	3. Solving Linear and Quadratic Equations of One Variable; *Hwk #2 Due*
21-Sep	4. Solving Equations of Two Variables; *Hwk #3 Due*
28-Sep	NO CLASSES – Instructor Holiday
5-Oct	5. Creating Equations: Polynomials, Exponents & Logarithms *Hwk #4 Due*
12-Oct	6. Functions, Graphing, Exponents and Logarithms; *Hwk #5; Exam #1*
19-Oct	7. Logarithmic and Exponential Models; *Hwk #6; Exam #1 Review*
26-Oct	8. Trigonometric Functions, Pythagorean Theorem; *Hwk #7 Due*
2-Nov	9. Applications of Trigonometry; *Hwk #8 Due*
9-Nov	10. Analytic Trigonometry: Identities & Graphing; *Hwk #9 Due; Exam #2*
16-Nov	11. Areas and Volumes of Geometric Solids; *Hwk #10; Exam #2 Review*
23-Nov	12. Systems of Equations and Inequalities
30-Nov	13. Series and Sequences, Review topics
7-Dec	Final Examination *Exam Emailed Monday 9:00am - Due at 1:00pm ET ?*

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www.CharlesRubenstein.com/150

20fa10.pdf = *This slide set**

20fa10_h.pdf = *slides as 6-up handouts**

**My goal is to post these not later than Noon on the Friday or Saturday before our Zoom Class Meetings*

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Questions?

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This Class Session #10

Class Sessions Posted Online Friday before Class

***In class* – Session 10: Monday 9 November:**

- ***DUE:* Homework Set #09 by 12:00Noon 2 November!**
  - *NOTE: Quiz 9 = four problems from hwk*
- ***Review:* Homework Set #08; Textbook readings**
- ***Lecture: Applications and Models***

Exam #2 is a one hour exam with 20 @ 5 point questions that will be emailed after our Zoom Class on Monday 9 November (by 8:00pmET) - EXAM 2 is DUE by 11:00pm 9 November!

***In class* – Session 11: Monday 16 November:**

- ***DUE:* Homework Set #10 by 12:00Noon 9 November!**
  - *NOTE: Quiz 10 = four problems from hwk – LAST HOMEWORK!*
- ***Review: Exam 2; Homework Set #09; Textbook readings***
- ***Lecture: Areas and Volumes of Geometric Solids***

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Homework #8b.
REVIEW
Section 1.4 Problems

Section 1.4 (Factoring)
page 39 (page 37 in 5th Ed.):
Problem **82**

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Homework #8b Review

1.4 #82a Factor Completely.

$$\left\{\frac{n(n+1)}{2}\right\}^2 + (n+1)^3$$
$$= \frac{n^2(n+1)^2}{4} + (n+1)^3$$
$$= (n+1)^2\left[\frac{n^2}{4} + (n+1)\right]$$
$$= (n+1)^2\left(\frac{n^2}{4} + n + 1\right):$$
$$(n+1)^2\left(\frac{n}{4}+?\right)\left(\frac{n}{4}+?\right)$$
$$a \cdot c = \frac{1}{4}\cdot 1 = \frac{1}{4}$$
$$\frac{1}{2}\cdot\frac{1}{2} = \frac{1}{4} \qquad \frac{1}{2}+\frac{1}{2} = 1$$
$$(n+1)^2\left(\frac{n}{4}+\frac{1}{2}\right)\left(\frac{n}{4}+\frac{1}{2}\right)$$

$$(n+1)^2\left(\frac{n}{4}+\frac{1}{2}\right)\left(\frac{n}{4}+\frac{1}{2}\right)$$

Discard common factor $\frac{1}{2}$ out of both factors $\left(\frac{n}{4}+\frac{1}{2}\right)$

$$= (n+1)^2\left(\frac{n}{2}+1\right)\left(\frac{n}{2}+1\right)$$
$$= (n+1)^2\left(\frac{n}{2}+1\right)^2$$

final answer

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Homework #8b Review

1.4 #82b Factor Completely.

$$\frac{n(n+1)(2n+1)}{6} + (n+1)^2$$
$$= \frac{(n+1)}{6}\left[n(2n+1) + 6(n+1)\right]$$
$$= \frac{(n+1)}{6}(2n^2 + n + 6n + 6)$$
$$= \frac{(n+1)}{6}(2n^2 + 7n + 6):$$
$$\frac{(n+1)}{6}(2n\ \ )(2n\ \ )$$
$$a \cdot c = 2 \cdot 6 = 12$$
$$4 \cdot 3 = 12 \qquad 4 + 3 = 7$$
$$\frac{(n+1)}{6}(2n+4)(2n+3)$$

Discard common factor 2 out of $(2n+4)$

$$= \frac{(n+1)}{6}(n+2)(2n+3)$$

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Homework #8b Review

1.4 #82c Factor Completely.

$$\frac{1}{b}(a+bx)^2-\frac{a}{b}(a+bx)$$

$$=\frac{1}{b}(a+bx)\left[(a+bx)-a\right]$$

$$=\frac{1}{b}(a+bx)(bx)$$

$$=x(a+bx)$$

final answer

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Homework #8c.
REVIEW
Section 1.5 Problems

Section 1.5 (Rational Expressions)
pages 47-48 (page 45 in 5th Ed.):
Problems **34, 36, 38, 40, 42 , 54, 56, 58, 60, 62**

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Homework #8c Review

1.5 #34. Perform the indicated operations and simplify:

$$\frac{1}{x-1}+\frac{2}{x-2} \quad =\frac{1}{(x-1)}\cdot\frac{(x-2)}{(x-2)}+\frac{2}{(x-2)}\cdot\frac{(x-1)}{(x-1)}$$

$$=\frac{x-2}{(x-1)(x-2)}+\frac{2x-2}{(x-1)(x-2)}$$

$$=\frac{x-2+2x-2}{(x-1)(x-2)}$$

$$=\frac{3x-4}{(x-1)(x-2)}$$

final answer

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Homework #8c Review

1.5 #36. Perform the indicated operations and simplify:

$$\frac{a}{8b}-\frac{b}{12a} \quad =\frac{a}{8b}\cdot\frac{(3a)}{(3a)}-\frac{b}{12a}\cdot\frac{(2b)}{(2b)}$$

$$=\frac{3a^2}{24ab}-\frac{2b^2}{24ab}$$

$$=\frac{3a^2-2b^2}{24ab}$$

final answer

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Homework #8c Review

1.5 #38. Perform the indicated operations and simplify:

$$\frac{4x-1}{6x^3}+\frac{2}{3x^2} \quad =\frac{4x-1}{6x^3}+\frac{2}{3x^2}\cdot\frac{(2x)}{(2x)}$$

$$=\frac{4x-1}{6x^3}+\frac{4x}{6x^3}$$

$$=\frac{4x-1+4x}{6x^3}$$

$$=\frac{8x-1}{6x^3}$$

final answer

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Homework #8c Review

1.5 #40. Perform the indicated operations and simplify:

$$\frac{x}{x-y}-\frac{y}{x+y} \quad =\frac{x}{x-y}\cdot\frac{(x+y)}{(x+y)}-\frac{y}{x+y}\cdot\frac{(x-y)}{(x-y)}$$

$$=\frac{x^2+xy}{(x-y)(x+y)}-\frac{xy-y^2}{(x-y)(x+y)}$$

$$=\frac{x^2+xy-(xy-y^2)}{(x-y)(x+y)}$$

$$=\frac{x^2+xy-xy+y^2}{(x-y)(x+y)}$$

$$=\frac{x^2+y^2}{(x-y)(x+y)}$$

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Homework #8c Review

1.5 #42. Perform the indicated operations and simplify:

$$\frac{4}{r}-\frac{3}{r+2} = \frac{4}{r}\cdot\frac{(r+2)}{(r+2)}-\frac{3}{r+2}\cdot\frac{(r)}{(r)}$$
$$=\frac{4r+8}{r(r+2)}-\frac{3r}{r(r+2)}$$
$$=\frac{4r+8-3r}{r(r+2)}$$
$$=\frac{r+8}{r(r+2)}$$

final answer

Homework #8c Review

1.5 #54. Simplify the complex fraction and perform all indicated operations

$$\frac{1-\frac{r^2}{s^2}}{1+\frac{r}{s}} = \frac{1-\frac{r^2}{s^2}}{1+\frac{r}{s}}\cdot\frac{(s^2)}{(s^2)}$$
$$=\frac{s^2-r^2}{s^2+rs}$$
$$=\frac{(s+r)(s-r)}{s(s+r)}$$
$$=\frac{s-r}{s}$$

final answer

Homework #8c Review

1.5 #56. Simplify the complex fraction and perform all indicated operations

$$\frac{\frac{a}{a-b}-\frac{b}{a+b}}{a^2-b^2} = \frac{\frac{a}{a-b}-\frac{b}{a+b}}{(a+b)(a-b)}\cdot\frac{(a+b)(a-b)}{(a+b)(a-b)}$$
$$=\frac{a(a+b)-b(a-b)}{(a+b)^2(a-b)^2}$$
$$=\frac{a^2+ab-ab+b^2}{(a+b)^2(a-b)^2}$$
$$=\frac{a^2+b^2}{(a+b)^2(a-b)^2}$$

final answer

Homework #8c Review

1.5 #58. Simplify the complex fraction and perform all indicated operations

$$\frac{\frac{4}{x^2-4}+1}{\frac{x}{x^2+x-6}} = \frac{\frac{4}{(x+2)(x-2)}+1}{\frac{x}{(x+3)(x-2)}}\cdot\frac{(x+3)(x+2)(x-2)}{(x+3)(x+2)(x-2)}$$
$$=\frac{4(x+3)+(x+3)(x+2)(x-2)}{x(x+2)}$$
$$=\frac{(x+3)[4+(x+2)(x-2)]}{x(x+2)}$$
$$=\frac{(x+3)(4+x^2-4)}{x(x+2)}$$
$$=\frac{x^2(x+3)}{x(x+2)}$$

final answer $=\frac{x(x+3)}{(x+2)}$

Homework #8c Review

1.5 #60. Simplify the complex fraction and perform all indicated operations

$$\frac{\frac{x}{x-2}-\frac{x}{x+2}}{\frac{2x}{x-2}+\frac{x^2}{x-2}} = \frac{\frac{x}{x-2}-\frac{x}{x+2}}{\frac{2x}{x-2}+\frac{x^2}{x-2}}\cdot\frac{(x+2)(x-2)}{(x+2)(x-2)}$$
$$=\frac{x(x+2)-x(x-2)}{2x(x+2)+x^2(x+2)}$$
$$=\frac{x^2+2x-x^2+2x}{(x+2)(2x+x^2)}$$
$$=\frac{4x}{(x+2)x(2+x)}$$

final answer $=\frac{4}{(x+2)^2}$

Homework #8c Review

1.5 #62 Simplify the complex fraction and perform all indicated operations

$$2+\frac{3}{1+\frac{2}{1-x}} = 2+\left[\frac{3}{\left(1+\frac{2}{1-x}\right)}\cdot\frac{(1-x)}{(1-x)}\right]$$
$$=2+\frac{3-3x}{(1-x)+2}$$
$$=2+\frac{3-3x}{3-x}$$
$$=\frac{2}{1}\frac{(3-x)}{(3-x)}+\frac{3-3x}{3-x}$$
$$=\frac{6-2x}{3-x}+\frac{3-3x}{3-x}$$
$$=\frac{6-2x+3-3x}{3-x}$$

final answer $=\frac{9-5x}{3-x}$

Questions?

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In Chapter 7

7. The Trigonometric Functions

- **7.1 Angles and Their Measurement**
- **7.2 Right Triangle Trigonometry**
- **7.3 The Trigonometric Functions**
- **7.4 Special Values and Properties of Trigonometric Functions**
- **7.5 Graphs of the Trigonometric Functions**
- **7.6 Graphs: Amplitude, Period and Phase Shift**
- **7.7 The Inverse Trigonometric Functions**
- **7.8 Applications and Models**

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Chapter 7

Applications and Models

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Applications Involving Right Triangles

In this section, the three **angles** of a right triangle are denoted by the letters ***A***, ***B***, and ***C*** (where ***C*** is the *right angle*), and the **lengths of the sides** opposite these angles by the letters ***a***, ***b***, and ***c***, respectively (where ***c*** is the *hypotenuse*).

Word Problem:

A safety regulation states that the maximum angle of elevation for a rescue ladder is 72°

A fire department's longest ladder is 110 feet.

What is the maximum safe rescue height?

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Finding a Side of a Right Triangle

A safety regulation states that the maximum angle of elevation for a rescue ladder is 72°- A fire department's longest ladder is 110 feet. What is the maximum safe rescue height?

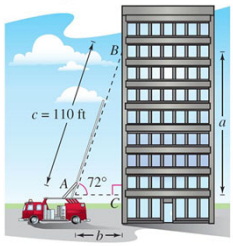
Solution:

- See the sketch
- From the Equation $\sin A = \frac{a}{c}$,

it follows that $a = c \sin A$

$= 110 \sin 72°$

≈ 104.6



So, the maximum safe rescue height is about 104.6 feet above the height of the fire truck.

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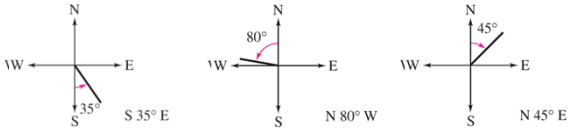
Trigonometry and Bearings

In surveying and navigation, directions can be given in terms of **bearings.**

A bearing measures the acute angle that a path or line of sight makes with a fixed north-south line.

For instance, the bearing **S 35° E**, shown below, means ***35 degrees east of south.***



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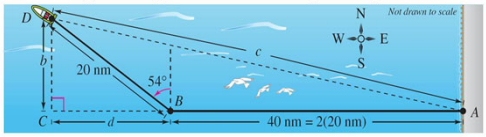
Finding Directions in Terms of Bearings

A ship leaves port at noon and heads **due west** at 20 knots, or 20 nautical miles (nm) per hour. At 2 P.M. the ship changes course to **N 54° W**, as shown below. Find the ship's bearing and distance from the port of departure at 3 P.M.



For triangle *BCD*, you have $\boldsymbol{B = 90° - 54°}$

The two sides of this triangle can be determined to be

$$\boldsymbol{b = 20 \sin 36° \text{ and } d = 20 \cos 36°}$$

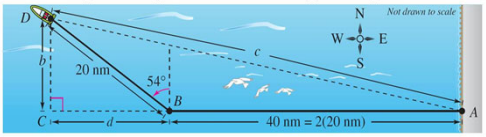
For triangle *ACD*, you can find angle *A* as follows.

$$\tan A = \frac{b}{d + 40} = \frac{20 \sin 36°}{20 \cos 36° + 40} \approx 0.2092494$$

and $A \approx \arctan 0.2092494 \approx 0.2062732$ radian $\approx 11.82°$

Finding Directions in Terms of Bearings

A ship leaves port at noon and heads **due west** at 20 knots, or 20 nautical miles (nm) per hour. At 2 P.M. the ship changes course to **N 54° W**, as shown below. Find the ship's bearing and distance from the port of departure at 3 P.M.



The angle with the north-south line is 90°− 11.82°= 78.18°.

So, the bearing of the ship is **N 78.18° W**.

Finally, from triangle *ACD*, you have $\sin A = \frac{b}{c}$,

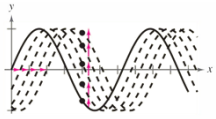
$$c = \frac{b}{\sin A} = \frac{20 \sin 36°}{\sin 11.82°} \approx 57.4$$

The ship is 57.4 nautical miles from the port.

Sine Waves and Harmonic Motion

One illustration of the relationship between sine waves and harmonic motion is in the wave motion that results when a stone is dropped into a calm pool of water. The waves move outward in roughly the shape of sine (or cosine) waves, as shown here

Similarly, if you are fishing and your fishing bobber is attached so that it does not move horizontally, as the waves move outward from the dropped stone, your fishing bobber will move up and down in simple harmonic motion, as shown here.



Simple Harmonic Motion

Definition of Simple Harmonic Motion

A point that moves on a coordinate line is in **simple harmonic motion** when its distance d from the origin at lime t is given by either

$$d = a \sin \omega t \quad \text{or} \quad d = a \cos \omega t$$

where a and ω are real numbers such that $\omega > 0$.

The motion has amplitude $|a|$, period $\frac{2\pi}{\omega}$, and frequency $\frac{\omega}{2\pi}$.

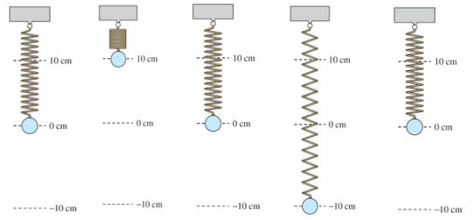
If the spring is at equilibrium ($d = 0$) when $t = 0$,

we use the equation:

$$d = a \sin \omega t$$

Harmonic Motion

Consider a ball that is bobbing up and down on the end of a spring, as shown in the Figures below, in a sine wave motion:



Equilibrium | **Maximum positive displacement** | **Equilibrium** | **Maximum negative displacement** | **Equilibrium**

Harmonic Motion -1

Suppose that 10 centimeters is the maximum distance the ball moves vertically upward or downward from its equilibrium (at rest) position.

Suppose further that the time it takes for the ball to move from its maximum displacement above zero to its maximum displacement below zero and back again is $t = 4$ **seconds**.

Assuming the ideal conditions of perfect elasticity and no friction or air resistance, the ball would continue to move up and down in a uniform and regular manner.

From this spring you can conclude that the period (***time for one complete cycle***) of the motion is

Period = 4 seconds

Harmonic Motion -2

Suppose that 10 centimeters is the maximum distance the ball moves vertically upward or downward from its equilibrium (at rest) position.

Assuming the ideal conditions of perfect elasticity and no friction or air resistance, the ball would continue to move up and down in a uniform and regular manner.

its amplitude (maximum displacement from equilibrium) is

Amplitude = 10 centimeters

and its **frequency** (number of cycles per second) is

$$\text{Frequency} = \frac{1}{4} \text{ cycle per second.}$$

Motion of this nature can be described by a sine or cosine function and is called **simple harmonic motion**

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Harmonic Motion - 3

$d = a\sin\omega t$ or $d = a\cos\omega t$

When the maximum distance the ball moves vertically upward or downward from its equilibrium position is 10 cm and the period 4 seconds:

The equation of motion is calculated from

$$\text{Amplitude} = |a| = 10 \qquad \text{Period} = \frac{2\pi}{\omega} = 4 \quad \Rightarrow \quad \omega = \frac{\pi}{2}$$

$$d = 10\sin\frac{\pi}{2}t.$$

and the Frequency $= \frac{\omega}{2\pi} = \frac{\pi/2}{2\pi} = \frac{1}{4}$ cycles per second.

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Questions?

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Next Class Session #11

***In class* – Session 11: Monday 16 November:**

- ***DUE:** Homework Set #10 by 12:00Noon 16 November!*
  NOTE: Quiz 10 = four problems from hwk – LAST HOMEWORK!
- ***Review: Exam 2; Homework Set #09; Textbook readings***
- ***Lecture: Areas and Volumes of Geometric Solids***

***In class* – Session 12: Monday 23 November:**

- ***DUE:** Homework Set #10 was the last homework assignment…*
- ***Review:** Homework Set #10; Textbook readings*
- ***Lecture: Systems of equations and Inequalities***

***In class* – Session 13: Monday 30 November:**

- ***Review:** Textbook readings*
- ***Lecture: Series and Sequences,***
- ***Final Review Questions (must be submitted in advance)***

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About Exam #2 – Worth 20%

Exam #2 is a one hour exam with 20 @ 5 point questions that will be emailed after our Zoom Class on Monday 9 November (by 8:00pmET)

EXAM 2 is DUE** **by 11:00pm 9 November!

HOW TO EMAIL ME YOUR EXAM:

1. You MUST use DARK BLACK pencil or pen on WHITE paper.
If I can't read your work you get a ZERO!

*2. Please scan **E2.docx** – THE ANSWER SHEET - as a PDF and save it as **lastname_E2.pdf** – **IF YOU CAN NOT SCAN –***
*Fill out the E2.docx file. Take a photo of any work unable to be 'typed out' and insert the photo(s) into the space allotted and save the file as: **lastname_E2.docx** and attach the file*
Include ONLY the Answer Sheet with any units… NO CLOUD LINKS!
*Email your file to me at: **crubenst@pratt.edu***
*With the Subject Line: **Math150 Exam***

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Any Questions?
Send me an email …

crubenst@pratt.edu
or
c.rubenstein@ieee.org

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End

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