Pratt

Math 150 – Fall 2020
Algebra & Trigonometry

Charles Rubenstein, Ph. D.
Professor of Engineering & Information Science

Session 11: Monday 11/16/20
6:30pm - 9:20pm
via REMOTE LEARNING
Revision 1

Instructor Contact Information

Dr. Charles Rubenstein <crubenst@pratt.edu>
Professor of Engineering & Information Science
Faculty Office: ARC G-49

Fall 2020 Virtual Office hours *ONLY*
Wednesdays 10:00am-2:00pm via Zoom Meeting
To make your appointment
Send me an email at least one day in advance :
crubenst@pratt.edu
or ***c.rubenstein@ieee.org***

 2

Math 150 – Class Topics

1. The Foundations of Algebra
2. Equations and Inequalities
3. Functions
4. Polynomial Functions
5. Rational Functions and Conic Sections
6. Exponential and Logarithmic Functions
7. The Trigonometric Functions
8. Analytic Trigonometry
9. Applications of Trigonometry
10. Systems of Equations and Inequalities
11. Matrices, Linear Systems, and Determinants
12. Topics in Algebra

 3

Draft Schedule: Math 150 – Fall 2020 – Remote Learning

Monday	Notes
24-Aug	1. Introduction: Numbers, Arithmetic Operations, Fractions
31-Aug	2. Manipulation of Algebraic Expressions; *Hwk #1 Due @ Noon*
7-Sep	NO CLASSES – Labor Day
14-Sep	3. Solving Linear and Quadratic Equations of One Variable; *Hwk #2 Due*
21-Sep	4. Solving Equations of Two Variables; *Hwk #3 Due*
28-Sep	NO CLASSES – Instructor Holiday
5-Oct	5. Creating Equations: Polynomials, Exponents & Logarithms *Hwk #4 Due*
12-Oct	6. Functions, Graphing, Exponents and Logarithms; *Hwk #5; Exam #1*
19-Oct	7. Logarithmic and Exponential Models; *Hwk #6; Exam #1 Review*
26-Oct	8. Trigonometric Functions, Pythagorean Theorem; *Hwk #7 Due*
2-Nov	9. Applications of Trigonometry; *Hwk #8 Due*
9-Nov	10. Analytic Trigonometry: Identities & Graphing; *Hwk #9 Due; Exam #2*
16-Nov	11. Areas and Volumes of Geometric Solids; *Hwk #10; Exam #2 Review*
23-Nov	12. Systems of Equations and Inequalities
30-Nov	13. Series and Sequences, Review topics
7-Dec	Final Examination *Emailed Monday by 6:00pm ET - Due at 10:00pm ET*

 4

www.CharlesRubenstein.com/150

20fa11.pdf = *This slide set**
20fa11_h.pdf = *slides as 6-up handouts**

**My goal is to post these not later than Noon on the Friday or Saturday before our Zoom Class Meetings*

 5

Questions?

 6

This Class Session #11

Exam #2 was a one hour exam with 20 @ 5 point questions that was emailed after our Zoom Class on Monday 9 November and was DUE by 11:00pm 9 November!

***In class* – Session 11: Monday 16 November:**

- ***DUE:* Homework Set #10 by 12:00Noon 16 November!**
  - **NOTE: Quiz 10 = four problems from hwk = LAST HOMEWORK!**
- ***Review: Exam 2*; Homework Set #09; Textbook readings**
- ***Lecture: Areas and Volumes of Geometric Solids***

***In class* – Session 12: Monday 23 November:**

- ***Review: Homework Set #10; Textbook readings***
- ***Lecture: Systems of Equations and Inequalities***

***In class* – Session 13: Monday 30 November:**

- ***Review: Textbook readings***
- ***Lecture: Series and Sequences; Review Topics***

Monday 7 December by 6:30pm FINAL EXAM Emailed
MUST BE RETURNED BY 10:30pm

About Exam #2 – Worth 20%

Exam #2 is a one hour exam with 20 @ 5 point questions that will be emailed after our Zoom Class on Monday 9 November (by 8:00pmET)

***EXAM 2 is DUE* by 11:00pm 9 November!**

HOW TO EMAIL ME YOUR EXAM:

1. You MUST use DARK BLACK pencil or pen on WHITE paper.
If I can't read your work you get a ZERO!

2. Please scan your work as a PDF and save it as ***lastname_E2.pdf***
HOWEVER – IF YOU CAN NOT SCAN –
Fill out the docx file. Take a photo of any work unable to be 'typed out' and insert the photo(s) into the space allotted and save the file as: ***lastname_E2.docx*** *and attach the file (**NO CLOUD LINKS**)*
Include the worked out problems AND solutions AND any units…

Email your file to me at: ***crubenst@pratt.edu***
With the Subject Line: ***Math150 Exam***

Exam 2 Review

EXAM 2 STATISTICS:

LOW GRADE: 58
HIGH GRADE: 98
Average Grade: 82%

14% lower than first exam!

In the past 25% or more of class did *poorly* on Questions:

3, 5, 6, 10, 14, 16, 17, 18, 19 and 20

Exam 2 Review

In problems 1 through 3, reduce the expression to a single fraction. (For example: ½ + ¼ = ¾)

1. $\frac{1}{3} + \frac{4}{7} =$

$$\frac{7}{7\cdot3} + \frac{3\cdot4}{3\cdot7} = \frac{19}{21}$$

Exam 2 Review

In problems 1 through 3, reduce the expression to a single fraction. (For example: ½ + ¼ = ¾)

2. $\frac{a}{3} + \frac{b}{7}$

$$\frac{7a}{7\cdot3} + \frac{3b}{3\cdot7} = \frac{7a+3b}{21}$$

Exam 2 Review

In problems 1 through 3, reduce the expression to a single fraction. (For example: ½ + ¼ = ¾)

3. $\frac{a}{3} + \left(\frac{7}{b-a}\right)^2 =$

Easy way: $\frac{a(b-a)^2}{3(b-a)^2} + \frac{3\cdot7^2}{3(b-a)^2} = \frac{a(b-a)^2+147}{3(b-a)^2}$

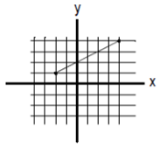
also = $a/3 + 49/(b-a)^2$

Hard way: $a(b-a)(b-a) = a(b^2 - 2ab + a^2)$ and thus:

$$\frac{(ab^2 - 2a^2b + a^3) + 147}{3b^2 - 6ab + 3a^2}$$

Exam 2 Review

4a. (**2 points**) Sketch the points
(-2, 1) and **(4, 4)**



4b. (**3 points**) Find the midpoint between these two points.

$$\left(\frac{-2+4}{2}, \frac{4+1}{2}\right) = \left(1, \frac{5}{2}\right) = (1, 2.5)$$

 13

Exam 2 Review

5a. **(3 points)** Write the equation of a line that has **slope 2** and passes through the point **(0, 5)**

$$y - y_1 = m(x - x_1)$$
$$y - 5 = 2(x - 0)$$
$$y = 2x + 5$$

5b. (2 points) Check that the point $x=0$ and $y=5$ satisfies your equation.

If you noticed that ***(0, 5)*** *is the* ***y-intercept****, you could have immediately written:*

$$y = mx + b \text{ or } y = 2x + 5$$

at (0,5):

$$5 = 2(0) + 5 \text{ or } 5 = 5$$

 14

Exam 2 Review

6. Given $W = \frac{2PR}{R-r}$ solve for r

Cross multiplying gives $(R-r)W = 2PR$ or $RW - rW = 2PR$,
Rearranging, we get $rW = RW - 2PR$. Dividing by W, we end up with
$r = (RW - 2PR)/W$ or $r = R(1 - 2P/W)$.

OR: $r = R - (2PR)/W$

OR: $r = R(W - 2P)/W$

OR: $r = R(1 - 2P/W)$

 15

Exam 2 Review

7. Find the **distance** between the points **(1,1)** and **(5,4)** and the **slope** of the line that passes through these points.

Distance Calculation: (**3 points**)

$$D = \sqrt{(5-1)^2 + (4-1)^2} = \sqrt{16+9} = \sqrt{25} = 5$$

Slope Calculation: (**2 points**)

$$m = \left(\frac{4-1}{5-1}\right) = \frac{3}{4}$$

 16

Exam 2 Review

8. Find the **equation of the line** that passes through the points: **(3, 2)** and **(6,4).**

$$m = \left(\frac{4-2}{6-3}\right) = \frac{2}{3} \qquad y - y_1 = m(x - x_1)$$

$$y - 2 = \frac{2}{3}(x-3), \quad y - 2 = \frac{2x}{3} - 2, \quad y = 2\frac{x}{3}$$

Or: y = 2/3 x

Or: y = 0.67 x

 17

Exam 2 Review

9. Find the **values of** x that satisfy the equation:
$x^2 + 2x - 8 = 0$.

a. By Factoring: $(x+4)(x-2) = 0$
therefore $x=-4$ **or** $x=+2$

b. By Completing the square: $x^2 + 2x = 8$
$(x+1)^2 = 8 + 1^2$; $x+1 = \sqrt{9}$; $x = -1 \pm 3$; $x = -4$ **or** $x = +2$

c. By Quadratic Formula:
$x = (-2 \pm \sqrt{(4+32)})/2 = (-2 \pm 6)/2 = (-1 \pm 3)$

$x = -4$ **or** $x = +2$

 18

Exam 2 Review

10. Find the **values of x** that solve the equation:

$x^2 + 2\pi x - 8 = 0$.

Using the standard Quadratic Equation with:

a = 1; b = 2π; c = -8

$$x = \frac{-2\pi \pm \sqrt{(2\pi)^2 - 4 \cdot -8}}{2} = -\pi \pm \sqrt{\pi^2 + 8}$$

$x = -\pi +/- \sqrt{(9.87+8)} = -\pi +/- \sqrt{(17.87)} = -\pi +/- 4.23$

$x = -3.14 + 4.23$ and $x = -3.14 - 4.23$

Thus: $x = +1.08565$ and $x = -7.36884$

 19

Exam 2 Review

11. Find the **equation of a line** that passes through the **point (0,1)** and is **perpendicular to the line y = 5x + 7**.

Slope = **5**, therefore perpendicular line slope = **-1/5**

$y - y_1 = m(x - x_1)$, $y - 1 = -(x-0)/5$, $y = 1 - x/5$

Or more accurately:

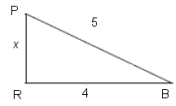
$y = -1/5\ x + 1$

 20

Exam 2 Review

12. *From his home, Ronald (**R**) would have to walk due **north** to get to his friend Paul's house (**P**) and due **east** to get to his friend Bianca's (**B**) house. It is **4 miles** from Ronald's house to Bianca's house and a straight-line distance of **5 miles** from Paul's house to Bianca's house. **How far is Ronald's house from Paul's house**?*

This is a right triangle, Pythagorean Theorem problem



$x^2 + 4^2 = 5^2$

$x^2 = 5^2 - 4^2$

$x^2 = 25 - 16$

$x^2 = 9$

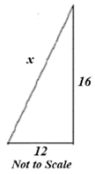
x = 3 miles

I took off 1 point from you if you left off or had the wrong units.

 21

Exam 2 Review

13. *A man is leaning a ladder against the side of his house to repair the roof. The top of the ladder reaches the roof, which is **16 feet** high. The base of the ladder is **12 feet** away from the house, where his son is holding it steady. How long is the ladder?*



Another ***Pythagorean Theorem problem…***

$12^2 + 16^2 = x^2$

$x^2 = 144 + 256$

$x^2 = 400$

x = 20 feet

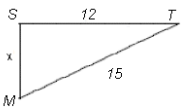
I took off 1 point from you if you left off or had the wrong units. *Not to Scale*

 22

Exam 2 Review

14. *On the school playground (**S**), the slide is due **west** of the tire swing (**T**) and due **south** of the monkey bars (**M**). If the distance between the slide and the tire swing is 12 **meters** and the distance between the tire swing and the monkey bars is 15 meters, how far is the slide from the monkey bars?*

Another ***Pythagorean Theorem problem…***



$x^2 + 12^2 = 15^2$

$x^2 = 15^2 - 12^2$

$x^2 = 225 - 144$

$x^2 = 81$

x = 9 meters

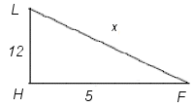
I took off 1 point from you if you left off or had the wrong units.

 23

Exam 2 Review

15. *Leslie (**L**) is fishing from a small boat. Her fishing hook (**H**) is **12 meters** below her, and a fish (**F**) is swimming at the same depth as the hook, **5 meters** away. How far away is Leslie from the fish?*

Another ***Pythagorean Theorem problem…***



$12^2 + 5^2 = x^2$

$x^2 = 144 + 25$

$x^2 = 169$

x = 13 meters

I took off 1 point from you if you left off or had the wrong units.

 24

Exam 2 Review

16. *A film shop carrying black-and-white film and color film has $4000 in inventory. The profit on black-and-white film is 12%, and the profit on color film is 21%. If all the film is sold, and if the profit on color film is $150 less than the profit on black-and-white film, how much was invested in each type of film?*

Let BW Sales = x; *then* Color = $(4000 - x)$

Equation is therefore: $0.21(4000 - x) = 0.12(x) - 150$

multiply both sides by 100: $21(4000 - x) = 12(x) - 15000$

$84{,}000 - 21x = 12x - 15{,}000$

$-33x = -99{,}000$

$x = 3{,}000$

$3,000 was invested in BW film,

and $4{,}000 - x$ = **$1,000 invested in Color Film.**

I took off 1 point from you if you left off or had the wrong units.

 25

Exam 2 Review

17. *How many ounces of Ceylon tea worth $1.50 per ounce and how many ounces of Formosa tea worth $2.00 per ounce must be mixed to obtain a mixture of 8 ounces that is worth $1.85 per ounce?*

Let x = ounces Ceylon Tea; *thus* $(8 - x)$ = ounces Formosa Tea

$1.50x + 2.00(8 - x) = 1.85\ (8)$

$1.50x + 16 - 2x = 14.8$

$-0.5x = -1.2$

x = **2.4 ounces of Ceylon Tea**

Therefore ***8.0 - x = 5.6 ounces Formosa Tea***

For some UNKNOWN reason a large number of you indicated 8-2.4 = 5.2 Oz…
I took off 1 point from you if you answered 5.2oz or if you left off 'oz'.
If you also had a decimal problem (e.g., 52 oz) I took off 2 points.

 26

Exam 2 Review

18. *Find the **width of a strip** that has been mowed around a **rectangular field 60 feet by 80 feet** if one half of the lawn has not yet been mowed.*

Let width of strip = x *and field* = *(60)(80) = 4800 square feet*

$(60 - 2x)(80 - 2x) = ½\ (60)(80)$ = ½ (4800) = 2400

$(60 - 2x)(80 - 2x) =$ 2400

(FOIL) $4800 - 120x - 160x + 4x^2 = 2400$

$4x^2 - 280x + 4800 = 2400$;

dividing both sides by 4: $x^2 - 70x + 1200 = 600$

$x^2 - 70x + 600 = 0$:

$(x - 60)(x - 10) = 0$; x *cannot be 60*. **Width = 10 feet**

I took off 1 point from you if you left off or had the wrong units.

 27

Exam 2 Review

19. *The surface area of a cube is 294 square inches. What is the length of each edge of the cube?*

Let length of an edge = x

Governing Equation: $S = 6x^2$

$294 = 6x^2$

$49 = x^2$

$x = 7$

each edge is 7 inches long

I took off 1 point from you if you left off or had the wrong units.

 28

Exam 2 Review

20. *A carpet factory manufactures bolts of carpet **10 feet wide**. A **large bolt of carpet covers 8 linear feet more than a small** bolt of carpet. If the **large bolt of carpet covers at most 200 square feet** of floor, what is the **largest length of a small bolt**?*

Let length of small bolt = x; *thus* length of large bolt = $x + 8$

Equation: $10(x + 8) \le 200$

$x + 8 \le 20$

$x \le 12$ feet is largest length of a small bolt

I took off 1 point from you if you left off or had the wrong units.

 29

Questions?

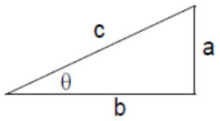
 30

Homework #9
REVIEW
Triangle Functions

 31

Homework #9 Review

1. Write expressions for sin(**θ**), cos(**θ**), and tan(**θ**) in terms of sec(**θ**).



Since sec(**θ**) = hyp / adj = 1/ cos(**θ**);

sin(**θ**) = opp/hyp = tan (**θ**) /sec(**θ**); [proof = (o/a) / (h/a) = o/h]

cos(**θ**) = adj/hyp = 1/sec(**θ**) ; [proof = 1 / (h/a) = a/h]

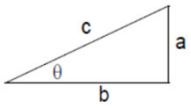
tan(**θ**) = opp/adj = sin(**θ**) sec(**θ**); [proof = (o/h)(h/a) = o/a]

 32

Homework #9 Review

2. If a =1 and b =6, find the values of sin(***θ***), cos(***θ***), and tan(***θ***)

$c = \sqrt{a^2 + b^2} = \sqrt{1^2 + 6^2} = \sqrt{37} = 6.0828$



$\sin(\theta) = a/c = 1/\sqrt{37} = 0.1644$

$\cos(\theta) = b/c = 6/\sqrt{37} = 0.9864$

$\tan(\theta) = a/b = 1/6 = 0.1667$

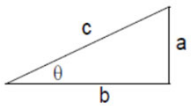
 33

Homework #9 Review

3. If c =1 and **θ** = 7 degrees, find a and b.

$\sin(7) = a/c = a = 0.1219$

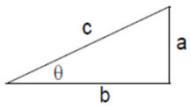
$\cos(7) = b/c = b = 0.9925$



 34

Homework #9 Review

4. If c =1 and **θ** = 1 radian. find a and b.



$\sin(\boldsymbol{1r}) = a/c = a = apx\ 0.8387\text{-}\ (for\ 0.9948r)$

$\cos(\boldsymbol{1r}) = b/c = b = apx\ 0.5446\text{-}\ (for\ 0.9948r)$

 35

Homework #9 Review

5. If a =1 and b =6, find the value of **θ** in degrees and in radians.

$\sin(\theta) = a/c = 1/\sqrt{37} = 0.1645$ **(from part 1, above)**

from this calculation, using tables: Sin(*9*) = 0.1564; Sin(*10*) = 0.1736

extrapolating, 0.1736-0.1564 = 0.0172; 0.1645 – 0.1564 = 0.0081;

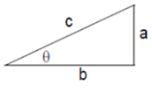
therefore

0.0081/0.0172 = 0.4709; thus add '9' to find ***θ*** = 9.47 degrees

θ = 9.47 degrees

radians = degrees * π/180 = 9.47 * 0.0175 = 0.1657 radians

θ = 0.1657 radians



 36

Homework #9 Review

For the next two examples use the following: *Area of a regular Polygon: where: S is the length of any side; N is the number of sides; π is PI, approximately 3.142; and TAN is the tangent function calculated in* ***radians:***

$$area = \frac{S^2 N}{4\tan\left(\frac{\pi}{N}\right)}$$

6. Find the area of a **regular hexagon** if the length of the sides is 1.

A = 1(6) / 4 [tan (*π/6)]*

= 6/4(tan(0.5236r))

= 6 / 4 (.5774)

= 6 / 2.3096

= 2.5979

Homework #9 Review

For the next two examples use the following: *Area of a regular Polygon: where: S is the length of any side; N is the number of sides; π is PI, approximately 3.142; and TAN is the tangent function calculated in* ***radians:***

$$area = \frac{S^2 N}{4\tan\left(\frac{\pi}{N}\right)}$$

7. Find the area of a **regular dodecagon** (12-sided polygon) if the length of the sides is 1.

A = 1(12) / 4 [tan (*π/12)]*

= 12 / 4 (tan(.2618))

= 12 / 4 (.2679)

= 12 /1.0716

= 11.1982

Homework #9 Review

8. Find the distance between the points **(1, 2, 6)** and **(4, 5, 9)** *(Use the 3-dimensional distance formula).*

A point is specified by an ordered triple of numbers called its coordinates. Let the coordinates of P_i *be* (x_i, y_i, z_i)*, for* $i = 1$ *and* 2*. The distance from* P_1 *to* P_2 *is equal to the distance from* P_2 *to* P_1*:*

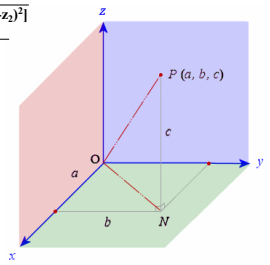
$$\mathbf{d = \sqrt{[(x_1-x_2)^2 + (y_1-y_2)^2 + (z_1-z_2)^2]}}$$

Thus: $d = \sqrt{[(4-1)^2 + (5-2)^2 + (6-9)^2]}$

$= \sqrt{[(3)^2 + (3)^2 + (3)^2]}$

$= \sqrt{[\,9 + 9 + 9\,]}$

$\mathbf{d = \sqrt{27} = 5.1962}$



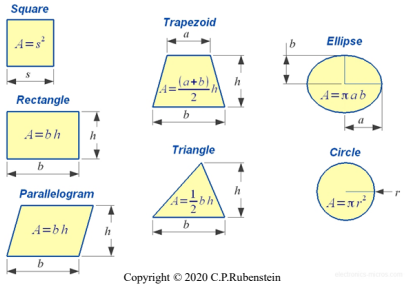
Questions?

Session 11

Areas and Volumes of Geometric Solids

What is Area?

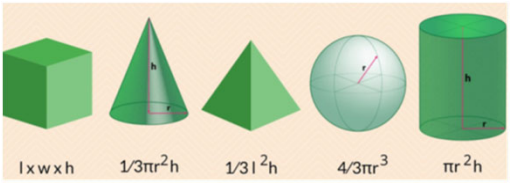
An area is a quantity that expresses the extent of a two-dimensional figure or shape or planar lamina in the plane. Lamina shapes include 2D figures that can be drawn on a plane, e.g., circle, square, triangle, rectangle, trapezium, rhombus and parallelogram. **Area of shapes** such as circle, triangle, square, rectangle, parallelogram, etc. are the region occupied by them in space.



What is Volume?

Volume

is the Three-dimensional space occupied by a Solid, Liquid, or Gas



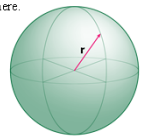
The SI unit of volume is the cubic meter (m^3), but many units exist. Other volume units include the liter (L), cubic centimeter (cm^3), fluid ounce (fl oz), pint (pt), quart (qt), gallon (gal), teaspoon (tsp), and tablespoon (tbls).

 43

Surface Areas and Volumes - 1

a. Sphere Surface Area and Volume Formula

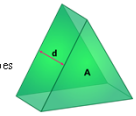
A sphere is a solid figure where every point on the surface is equidistant from the center of the sphere. This distance is the radius, **r**, of the sphere.



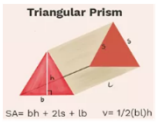
Surface area = $4\pi r^2$
Volume = $\frac{4}{3}\pi r^3$

b. Prism Surface Area and Volume Formula

A prism is a geometric shape consisting of a stack of identical base shapes stacked on top of each other to a depth **d**.
This prism is a prism formed by a stack of triangles.



Surface Area of a Prism
= 2 × (Area of the base shape)
+ (Perimeter of base shape) × (d)



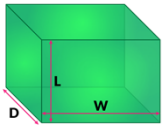
Volume of a Prism = (Area of base shape) × d

 44

Surface Areas and Volumes - 2

c. Box or Rectangular Prism Surface Area and Volume Formula

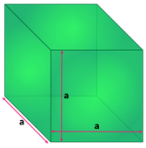
A box can be thought of a stack of rectangles **L** long and **W** wide piled on top of each other to a depth of **D**.
Surface Area of a Box = Sum of the areas of each face of the box, or



Surface Area of a Box = $2(L \times W) + 2(L \times D) + 2(W \times D)$
Volume of a Box = $L \times W \times D$

d. Cube Surface Area and Volume Formula

A cube is a special case box where all the sides are the same length.



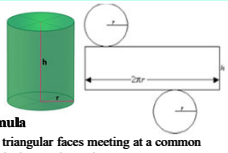
Surface Area of a Cube = $6a^2$
Volume of a Cube = a^3

 45

Surface Areas and Volumes - 3

e. Cylinder Surface Area and Volume Formula

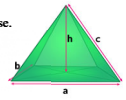
A cylinder is a prism where the base shape is a circle.



Surface Area of a Cylinder = $2\pi r^2 + 2\pi rh$
Volume of a Cylinder = $\pi r^2 h$

f. Square Pyramid Surface Area and Volume Formula

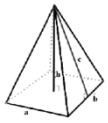
A pyramid is a solid shape consisting of a polygon base and triangular faces meeting at a common point above the base. A square pyramid is a pyramid where the base polygon is a square.
In the picture here, side a is the same length as side b.
All of the face triangles are isosceles triangles meeting at a point h above the base.



Surface Area = $a^2 + a\sqrt{a^2 + (2h)^2}$

Volume = $\frac{1}{3}a^2h$

g. For pyramids with identical face triangles (a = b = c)



Surface Area = $(1+\sqrt{3})\,a^2$

Volume = $\frac{\sqrt{2}}{6}a^3$

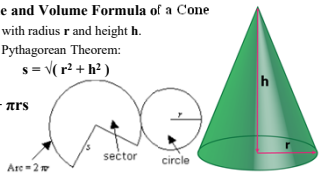
 46

Surface Areas and Volumes - 4

h. Surface Area Formula of a Cone and Volume Formula of a Cone

A cone is a pyramid with a circular base with radius **r** and height **h**.
The side length **s** can be found using the Pythagorean Theorem:

$s^2 = r^2 + h^2$ or $s = \sqrt{(r^2 + h^2)}$



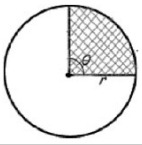
Surface Area of a Cone = $\pi r^2 + \pi rs$
Volume of a Cone = $\frac{1}{3}(\pi r^2 h)$

i. Area of a Circle Sector

The area of a circle sector can be calculated in degrees or radians.
We know that **$90^o = \pi/2$** radians and the central angle of the sector is θ

Then the ***area in radians*** $= \frac{\theta}{2} r^2$

And the ***area in degrees*** $= \frac{\theta}{360}\pi r^2$



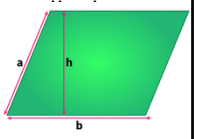
 47

Surface Areas and Perimeters - 1

a. Parallelogram Area and Perimeter Formula

A parallelogram is a closed figure formed by four sides and the opposite sides are parallel to each other. The 'height' (**h**) of a parallelogram is the distance from the measured side to its opposite parallel side.

Perimeter of a Parallelogram = 2a + 2b
Area of a Parallelogram = b · h



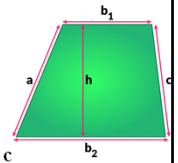
b. Rectangle Area and Perimeter Formula

A rectangle is a special parallelogram where the interior angles are all right angles.

Perimeter of a Rectangle = 2H + 2W
Area of a Rectangle = H · W

c. Trapezoid Area Area and Perimeter Formula

A trapezoid is another special quadrilateral (four-sided figure) where two of the sides are parallel. The 'height' (**h**) of a trapezoid is the distance between the two parallel sides.



Perimeter of a Trapezoid = $a + b_1 + b_2 + c$

Area of a Trapezoid = $\frac{1}{2}(b_1 + b_2) \cdot h$

 48

Surface Areas and Perimeters - 2

d. Ellipse Area and Perimeter Formula

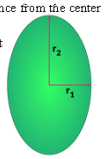
An ellipse is a closed figure where the path traced when the sum of the distances between two fixed points is a constant. The semi-minor axis of the oval is the shortest distance from the center of the ellipse (r_1) and the semi-major axis (r_2) is the longest distance from the center.

While the perimeter formula is rather complicated, and even the simplest approximation is not accurate:

$$p \approx 2\pi\sqrt{\frac{(r_1^2+r_2^2)}{2}}$$

the area formula is straightforward:

Area of an ellipse = $\pi r_1 r_2$



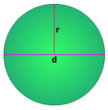
e. Circle Area and Perimeter Formula

A circle is a special ellipse where the semimajor and semiminor axes are the same size. All the points are the same distance from the center. This distance is known as the radius. The distance across the widest point of a circle is known as the diameter.

The perimeter of a circle is also known as the circumference.

Perimeter of a Circle = $2\pi r = \pi d$

Area of a Circle = πr^2



 49

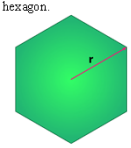
Surface Areas and Perimeters - 3

f. Hexagon Area and Perimeter Formula

A regular hexagon is a six-sided figure where each of the sides is of equal length. The length of these sides is equal to the distance from the center to the widest point of the hexagon.

Perimeter of a Hexagon = $6r$

Area of a Hexagon = $(3\sqrt{3})/2 \cdot r^2$

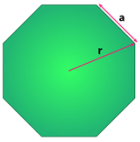


g. Octagon Area and Perimeter Formula

A regular octagon is an eight-sided figure with equal length sides.

Perimeter of an Octagon = $8a$

Area of an Octagon = $(2 + 2\sqrt{2})a^2$



 50

Surface Areas and Perimeters - 4

h. Triangle Area and Perimeter Formula

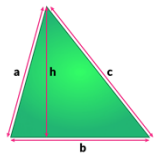
A triangle is a figure formed by three connected sides.

The perimeter is the sum of the lengths of the sides.

The 'height' (**h**) of a triangle is the highest point opposite of the side you choose as the base.

Perimeter of a Triangle = $a + b + c$

Area of a Triangle = $\frac{1}{2}b \cdot h$



 51

Questions?

 52

Next Class Session #12

Class Sessions Posted Online Friday before Class

***In class* – Session 12: Monday 23 November:**

- ***Review:* Homework Set #10; Textbook readings**
- ***Lecture: Systems of Equations and Inequalities***

Note: Pratt Closed for Thanksgiving

***In class* – Session 13: Monday 30 November:**

- ***Review:* Homework Set #10; Textbook readings**
- ***Lecture: Series and Sequences; Review Topics***

Monday 7 December
In-class FINAL EXAM
6:00pm – 10:00pm

 53

Any Questions?
Send me an email …

crubenst@pratt.edu
or
c.rubenstein@ieee.org

 54

End

 55