Pratt

Math 150 – Fall 2020
Algebra & Trigonometry

Charles Rubenstein, Ph. D.
Professor of Engineering & Information Science

Session 12: Monday 11/23/20
6:30pm - 9:20pm
via REMOTE LEARNING

Revision 1

Instructor Contact Information

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Fall 2020 Virtual Office hours *ONLY*
Wednesdays 10:00am-2:00pm via Zoom Meeting
To make your appointment
Send me an email at least one day in advance :
crubenst@pratt.edu
or ***c.rubenstein@ieee.org***

Math 150 – Class Topics

1. **The Foundations of Algebra**
2. **Equations and Inequalities**
3. **Functions**
4. **Polynomial Functions**
5. **Rational Functions and Conic Sections**
6. **Exponential and Logarithmic Functions**
7. **The Trigonometric Functions**
8. **Analytic Trigonometry**
9. **Applications of Trigonometry**
10. **Systems of Equations and Inequalities**
11. **Matrices, Linear Systems, and Determinants**
12. **Topics in Algebra**

Draft Schedule: Math 150 – Fall 2020 – Remote Learning

Monday	Notes
24-Aug	1. Introduction: Numbers, Arithmetic Operations, Fractions
31-Aug	2. Manipulation of Algebraic Expressions; *Hwk #1 Due @ Noon*
7-Sep	NO CLASSES – Labor Day
14-Sep	3. Solving Linear and Quadratic Equations of One Variable; *Hwk #2 Due*
21-Sep	4. Solving Equations of Two Variables; *Hwk #3 Due*
28-Sep	NO CLASSES – Instructor Holiday
5-Oct	5. Creating Equations: Polynomials, Exponents & Logarithms *Hwk #4 Due*
12-Oct	6. Functions, Graphing, Exponents and Logarithms; *Hwk #5; Exam #1*
19-Oct	7. Logarithmic and Exponential Models; *Hwk #6; Exam #1 Review*
26-Oct	8. Trigonometric Functions, Pythagorean Theorem; *Hwk #7 Due*
2-Nov	9. Applications of Trigonometry; *Hwk #8 Due*
9-Nov	10. Analytic Trigonometry: Identities & Graphing; *Hwk #9 Due; Exam #2*
16-Nov	11. Areas and Volumes of Geometric Solids; *Hwk #10; Exam #2 Review*
23-Nov	12. Systems of Equations and Inequalities
30-Nov	13. Series and Sequences, Review topics
7-Dec	Final Examination *Emailed Monday by 6:00pm ET - Due at 10:00pm ET*

www.CharlesRubenstein.com/150

20fa12.pdf = *This slide set**
20fa12_h.pdf = *slides as 6-up handouts**

**My goal is to post these not later than Noon on the Friday or Saturday before our Zoom Class Meetings*

Questions?

This Class Session #12

<u>*In class*</u> **– Session 12: Monday 23 November:**
- *Review:* ***Homework Set #10 & #11; Textbook readings***
- ***Lecture: Systems of Equations and Inequalities***

<u>*In class*</u> **– Session 13: Monday 30 November:**
- *Review:* ***Homework Set #12; Textbook readings***
- ***Lecture: Series and Sequences; Review Topics***

Monday 7 December
In-class "2-hour" FINAL EXAM
Emailed to the class not later than 6:00pm
Must be returned to me not later than 10:00pm

Questions?

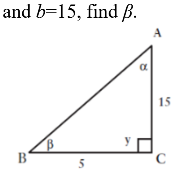
Homework #10a.
REVIEW
Section 7.8
(Special Values and Properties of Trigonometric Functions)

page 487 (page 481 in 5th Ed.):
Problems **2, 4, 6, 8, 10, 12, 14**

Homework #10 Review

Find the required part of the triangle *ABC* if the angle at "C" is 90 degrees.

Problem **2:** If a=5 and b=15, find β.

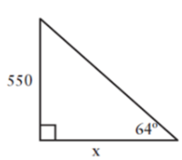


$$\tan\beta = \frac{15}{5} = 3$$

$$\beta = \tan^{-1}3 \approx 71.57° = 71°34'12''$$

Homework #10 Review

Problem **4:** A monument is 550 feet high. What is the length of the shadow cast by the monument when the sun is 64° above the horizon?



$$\tan 64° = \frac{550}{x}$$

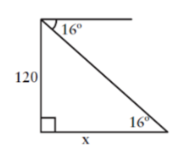
$$x = \frac{550}{\tan 64°} \approx 268.3$$

268.3 feet

Homework #10 Review

Problem **6:** A technician positioned on an oil-drilling rig 120 feet above the water spots a boat at an angle of depression of 16°.

How far is the boat from the rig?



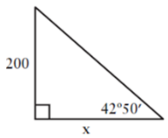
$$\tan 16° = \frac{120}{x}$$

$$x = \frac{120}{\tan 16°} \approx 418.5$$

418.5 feet

Homework #10 Review

Problem **8:** A hill is known to be 200 meters high. A surveyor standing on the ground finds the angle of elevation of the top of the hill to be 42º50′. Find the distance from the surveyor's feet to a point directly below the top of the hill.



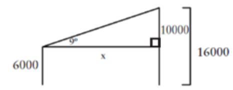
$$\tan 42^\circ 50' = \frac{200}{x}$$

$$x = \frac{200}{\tan 42^\circ 50'} \approx 215.73$$

215.73 meters

Homework #10 Review

Problem **10:** An airplane pilot wants to climb from an altitude of 6000 feet to an altitude of 16,000 feet. If the plane climbs at an angle of 9º with a constant speed of 22,000 feet per minute, how long will it take to reach the increased altitude?



$$\tan 9^\circ = \frac{10000}{x}$$

$$x = \frac{10000}{\tan 9^\circ} \approx 63137.5$$

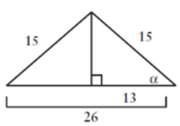
$$\frac{22000 \text{ feet}}{1 \text{ minute}} = \frac{63137.5 \text{ feet}}{t \text{ minutes}}$$

$$t = \frac{63137.5}{22000} \approx 2.87$$

2.87 minutes

Homework #10 Review

Problem **12:** The sides of an isosceles triangle are 15, 15, and 26 centimeters. Find the measures of the angles of the triangle. (Hint: The altitude of an isosceles triangle bisects the base.)



$$\cos\alpha = \frac{13}{15}$$

$$\alpha = \cos^{-1}\frac{13}{15} \approx 29.93^\circ = 29^\circ 55' 48''$$

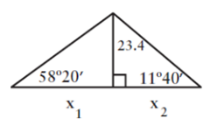
The two base angles are equal and each has a measure of 29º55′48″.

The third angle is

$180^\circ - 2(29^\circ 55' 48'') = 120^\circ 8' 24''$.

Homework #10 Review

Problem **14:** To determine the width of a river, markers are placed at each side of the river in line with the base of a tower that rises 23.4 meters above the ground. From the top of the tower, the angles of depression of the markers are 58º20′ and 11º40′. Find the width of the river



$$\tan 58^\circ 20' = \frac{23.4}{x_1}$$

$$x_1 = \frac{23.4}{\tan 58^\circ 20'} \approx 14.4$$

$$\tan 11^\circ 40' = \frac{23.4}{x_2}$$

$$x_2 = \frac{23.4}{\tan 11^\circ 40'} \approx 113.3$$

width of river $= x_1 + x_2 = 14.4 + 113.3 = 127.7$

127.7 meters

Homework #10b.

Review Exercises

(The Trigonometric Functions)

page 495 (page 485 in 5th Ed.):

Problems **2, 4**

Homework #10 Review

In Exercises 1–4, convert from degree measure to radian measure or from radian measure to degree measure.

Problem **2:** **3π / 2**

$$\frac{\frac{3\pi}{2}}{\pi} = \frac{\theta}{180}$$

$$\theta = \frac{180\left(\frac{3\pi}{2}\right)}{\pi} = 270^\circ$$

Homework #10 Review

Problem **4:** **45°**

$$\frac{\theta}{\pi} = \frac{45}{180}$$

$$\theta = \frac{45\pi}{180} = \frac{\pi}{4} \text{ radians}$$

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Questions?

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Homework #11a.
REVIEW
Section 7.4
(Special Values and Properties of Trigonometric Functions)
page 488
Problems **130, 132, 134, 136, 138 (trig identities)**

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Homework #11 Review

In Exercises 129–138, use trigonometric identities to transform the first expression into the second.

Problem **130:** $\frac{\cos t}{\sin t}, \quad \frac{1}{\tan t}$

$$\frac{\cos t}{\sin t} = \frac{\cos t}{\sin t}\cdot\frac{\frac{1}{\cos t}}{\frac{1}{\cos t}} = \frac{1}{\frac{\sin t}{\cos t}} = \frac{1}{\tan t}$$

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Homework #11 Review

Problem **132:** $\tan t \sin t + \cos t, \quad \frac{1}{\cos t}$

$$\tan t \sin t + \cos t = \left(\frac{\sin t}{\cos t}\right)(\sin t) + \cos t$$

$$= \frac{\sin^2 t}{\cos t} + \frac{\cos^2 t}{\cos t} = \frac{\sin^2 t + \cos^2 t}{\cos t}$$

$$= \frac{1}{\cos t}$$

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Homework #11 Review

Problem **134:** $\frac{1-\cos^2 t}{\sin t}, \quad \sin t$

$$\frac{1-\cos^2 t}{\sin t} = \frac{\sin^2 t}{\sin t} = \sin t$$

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Homework #11 Review

Problem **136:** $\dfrac{\cos^2 t}{1-\sin t}$, $\quad 1+\sin t$

$$\frac{\cos^2 t}{1-\sin t} = \frac{\cos^2 t}{1-\sin t}\cdot\frac{1+\sin t}{1+\sin t}$$

$$= \frac{\cos^2 t(1+\sin t)}{1-\sin^2 t}$$

$$= \frac{\cos^2 t(1+\sin t)}{\cos^2 t}$$

$$= 1+\sin t$$

Homework #11 Review

Problem **138:** $\dfrac{1}{1-\sin t}+\dfrac{1}{1+\sin t}$, $\quad \dfrac{2}{\cos^2 t}$

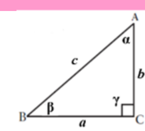
$$\frac{1}{1-\sin t}+\frac{1}{1+\sin t} = \frac{1+\sin t}{(1-\sin t)(1+\sin t)}+\frac{1-\sin t}{(1-\sin t)(1+\sin t)}$$

$$= \frac{1+\sin t+1-\sin t}{1-\sin^2 t}$$

$$= \frac{2}{\cos^2 t}$$

Homework #11b.
REVIEW
Section 9.1
(Trigonometric Identities and their Verification)
Law of Sines
page 534
Problems **2, 4, 6**

Law of Sines Review



$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$

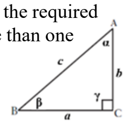
Or, alternatively…

$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c}$$

Homework #11 Review

In Exercises 1–12, use the law of sines to approximate the required part(s) of triangle ABC. Give both solutions if more than one triangle satisfies the given conditions.

Problem **2:** If $\alpha = 74^o$, $\gamma = 36^o$, and $c = 6.8$, find a



$$\frac{a}{\sin\alpha} = \frac{c}{\sin\gamma}$$

$$\frac{a}{\sin 74^o} = \frac{6.8}{\sin 36^o}$$

$$\frac{a}{0.9613} = \frac{6.8}{0.5878}$$

$$a = \frac{6.8\,(0.9613)}{0.5878} = \frac{6.5368}{0.5878}$$

$$a = 11.12$$

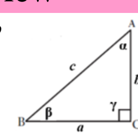
Homework #11 Review

Problem **4:** If $\alpha = 46^o$, $\beta = 88^o$, and $c = 10.5$, find b

First, find $\gamma = 180^o - (\alpha+\beta)$

$\gamma = 180^o - (46^o+88^o) = 180^o - 134^o$

$\gamma = 46^o$



$$\frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$

$$\frac{b}{\sin 88^o} = \frac{c}{\sin 46^o}$$

$$\frac{b}{0.9993} = \frac{10.5}{0.7193}$$

$$b = \frac{10.5\,(0.9993)}{0.7193} = \frac{10.4927}{0.7193} = 14.5873$$

Homework #11 Review

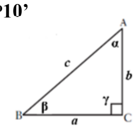
Problem **6:** If $\beta = 16^o30'$, $\gamma = 84^o40'$, and $a = 15$, find c

First find $\alpha = 180^o - (\beta + \gamma)$

$= 180^o - (16^o30' + 84^o40') = 180^o - (16^o + 84^o + [70'])$

$= 180^o - (16^o + 84^o + [1^o + 10']) = 180^o - 101^o10'$

$\alpha = 78^o\ [60'-10'] = 78^o50'$



$$\frac{a}{\sin\alpha} = \frac{c}{\sin\gamma}$$

$$\frac{a}{\sin 78^o50'} = \frac{c}{\sin 84^o40'}$$

$$c = \frac{15\sin 84^o40'}{\sin 78^o50'} = 15.22$$

Homework #11c. REVIEW

Section 9.2

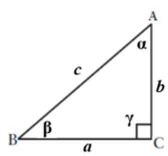
(Graphs of the Trigonometric Functions)

Law of Cosines

Page 542

Problems **2, 4, 6**

Law of Cosines Review



$$a^2 = b^2 + c^2 - 2bc\cos\alpha$$

$$b^2 = a^2 + c^2 - 2ab\cos\beta$$

$$c^2 = a^2 + b^2 - 2ab\cos\gamma$$

Homework #11 Review

In Exercises 1–10, use the law of cosines to approximate the required part of triangle ABC.

Problem 2: If **a = 5, b = 12 and c = 15; find** γ

Law of Cosines: $c^2 = a^2 + b^2 - 2ab\cos\gamma$

$$15^2 = 5^2 + 12^2 - 2(5)(12)\cos\gamma$$

$$-0.4667 = \cos\gamma$$

$$\gamma = \cos^{-1}(0.4667)$$

$$\gamma = 117.82^o = 117^o\ 49'\ 12''$$

Homework #11 Review

Problem **4:** If **b = 20, c = 13 and** $\alpha = 19^o\ 10'$**; find a.**

Law of Cosines: $a^2 = b^2 + c^2 - 2bc\cos\alpha$

$$a^2 = 20^2 + 13^2 - 2(20)(13)\cos 19^o10'$$

$$a^2 = 400 + 169 - 52\ [\cos 19^o10']$$

$$a^2 = 569 - 52\ [9.4458]$$

$$a^2 = 77.82$$

$$a = 8.82$$

Homework #11 Review

Problem **6:** If **a = 30, c = 40 and** $\beta = 122^o$**; find b.**

Law of Cosines: $b^2 = a^2 + c^2 - 2ac\cos\beta$

$$b^2 = 30^2 + 40^2 - 2(30)(40)\cos 122^o$$

$$b^2 = 900 + 1600 - 240\ [\cos 122^o]$$

$$b^2 = 2500 - 240\ [-0.52992]$$

$$b^2 = 2500 - [-127.1806]$$

$$b^2 = 3771.81$$

$$b = 61.42$$

Questions?

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Session 12
Chapter 10
Systems of Equations and Inequalities

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Systems of Equations

In the 'real' world we rarely get a simple equation presented to us…

Instead, there are relationships between various variables that often require more than a single equation to be solved that we MODEL by equations. One model uses 'polynomial curve fitting' which tries to match the data to a polynomial.

In general, to solve such problems we need to have at least one equation for each variable unknown:

One variable: one equation,
Two variables: two equations,
Three variables: three equations, etc.

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Equations in Two Unknowns

Consider the following word problem:
We have a pile of 9 coins consists of nickels and quarters.
If the total value of the coins is $1.25, how many of each type of coin are there?
Let **x** = the number of nickels and **y** = the number of quarters
The word problem can then be expressed as two equations:

$$x + y = 9 \text{ but also } 5x + 25y = 125$$

We have solved this type of equation set before by '**substitution**' recognizing if x + y = 9, then **y = 9 – x** and then solving the second equation:

$$5x + 25(9-x) = 125;\ 5x + 225 - 25x = 125$$
$$-20x = 125 - 225 \text{ or } 20x = 100;\ x = 5 \text{ and then } 9-x = 4 = y$$

This is an example of a system of equations.
We seek values of **x** and **y** that satisfy both equations. If we let **x = a** and **y = b** satisfy both equations, then the ordered pair (**a, b**) is called a solution of the system. In this case, the ordered pair would be: (5,4)

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Solving Equations by Substitution

Solve the system of equations by substitution:

$$x^2 + y^2 = 25 \quad \text{Eq.1}$$
$$x + y = -1 \quad \text{Eq.2}$$

We solve equation 2 for x: **x = -1 - y** and then substitute into Eq. 1:

$$(-1 - y)^2 + y^2 = 25 \quad \text{Eq.2a}$$

Note: $(-1 - y)^2 = (-1 - y)(-1 - y)$ ***using FOIL:***

$(-1 - y)(-1 - y) = (-1)(-1) + (-1)(- y) + (-y)(- 1) + (- y)(- y)$

$= 1 + (-y) + (-y) + y^2 = y^2 - 2y + 1$

Eq. 2a $[(-1 - y)^2] + y^2 = 25$ *becomes:* $[y^2 - 2y + 1] + y^2 = 25$

$2y^2 - 2y + 1 = 25$ or $2y^2 - 2y - 24 = 0$ dividing both sides by 2:

$y^2 - y - 12 = 0$; *using quadratic formula:*

$y = ½ [-(-1) +/- \sqrt{(-1)^2 +4(1)(-12)} = ½ [1 +/- 7/2] = y = 4, -3$

and thus **x = -5, 2**

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Systems of Linear Equations

In those cases where a system of equations *HAS one or more solutions*, it is termed a **CONSISTENT** system of equations

and

In those cases where a system of equations *DOES NOT have any solution*, it is termed an **INCONSISTENT** system of equations

A system consisting only of equations that are of the first degree in **x** and **y** is called a **system of linear equations**, or simply a **linear system**.

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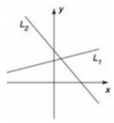
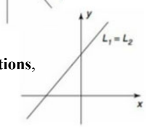
Systems of Linear Equations

When we graph a linear system of two equations on the same set of coordinate axes, there are three possibilities:

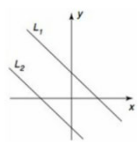
1. The two lines intersect at a point.
The system is **consistent** and has a **unique solution**, namely, the point of intersection.



2. The two equations are different forms of the same line.
The system is **consistent** and has an **infinite number of solutions**, namely, all points on the line.



3. The two lines are parallel.
Since the lines do not intersect, the system is **inconsistent** and **it has no solution**.



Solving by Elimination

The method of elimination seeks to combine the equations of a system in such a way as to eliminate one of the unknowns.

Let's return to our first problem where we deduced:

$$x + y = 9 \quad \text{Eq. 1}$$
$$5x + 25y = 125 \quad \text{Eq. 2}$$

If we multiply Eq.1 by **-5** we get:

$$-5x - 5y = -45$$

Adding this to Eq. 2 we remove the x from the resulting Eq. 3:

$$-5x - 5y = -45$$
$$5x + 25y = 125 \quad \text{Eq. 2}$$
$$20y = 80 \quad \text{Eq. 3}$$

from which we get **y = 4** and since **x + y = 9** : **x = 5**

Solving by Elimination

Let's solve the system of equations:

$$4x^2 + 9y^2 = 36 \quad \text{Eq. 1}$$
$$-9x^2 + 18y^2 = 4 \quad \text{Eq. 2}$$

If we multiply Eq.1 by **-2** we get:

$$-8x^2 - 18y^2 = -72$$

Adding this to Eq. 2 we remove the y from the resulting Eq. 3:

$$-17x^2 = -68$$
$$x^2 = 4 \quad \text{Eq. 3}$$
$$x = \pm 2$$

Substituting **x = 2** into Eq.1 we get

$4(2)^2 + 9y^2 = 36 \rightarrow 9y^2 = 36 - 4(2)^2$ *and* $y = \pm \frac{2}{3}\sqrt{5}$

Thus $x = \pm 2$ and $y = \pm \frac{2}{3}\sqrt{5}$

Word Problems

Three (**3**) vitamin pills and (**4**) herbal supplements cost **69** cents.

The same (**5**) vitamin pills and (**2**) herbal supplements cost **73** cents. What is the cost of each type of pill?

Let **x** = vitamin pill cost and **y** = herbal supplement pill cost. Then:

$$3x + 4y = 69 \quad \text{Eq.1}$$
$$5x + 2y = 73 \quad \text{Eq.2}$$

Multiply Eq.2 by **-2** and add to Eq.1 to eliminate **y**:

$$3x + 4y = 69$$
$$-10x - 4y = -146$$

Then: $-7x = -77$ *and* $x = 11$

Substituting **x = 11** into Eq.1 we solve for **y**:

$3(11) + 4y = 69 \rightarrow 4y = 36$ *and thus* $y = 9$

ANSWER: 11 cents *per vitamin and* **9 cents** *per supplement*

Systems of Equations

In some linear systems we find there are three unknowns…

Here, too, we can use the two techniques we have just studied:

substitution and **elimination**

to reduce these equations and find the unknowns.

A possible system of linear equations in three unknowns would be:

$$3x - y + 3z = -11 \quad \text{Eq.1}$$
$$2y + z = 2 \quad \text{Eq.2}$$
$$2z = -4 \quad \text{Eq.3}$$

(*Clearly this is a simple example as we can easily reduce this to a pair of equations in two unknowns as Eq. 3 simplifies immediately to z=-2*)

Equations with more than Two Unknowns

In this class we will NOT be doing 3-unknown linear systems…

However,

if we did want to solve linear equations in three unknowns we would be using techniques such as Gaussian Elimination to get the equations into 'triangular form' (Chapter 10.3) or using Matrices (Chapter 11) to solve for the three unknowns…

Questions?

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Next Class Session #13

Class Sessions Posted Online Friday before Class

Note: Pratt Closed for Thanksgiving

In class – **Session 13: Monday 30 November:**

- *Review: Homework Set #12; Textbook readings*
- *Lecture: Series and Sequences; Review Topics*

Monday 7 December

In-class "2-hour" FINAL EXAM

Emailed to the class not later than 6:00pm

Must be returned to me not later than 10:00pm

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Any Questions?
Send me an email …

crubenst@pratt.edu
or
c.rubenstein@ieee.org

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End

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