


**Pratt** 

**Math 150 – Fall 2021  
Algebra & Trigonometry**  
Charles Rubenstein, Ph. D.  
Professor of Information Science

**Session 2: Monday 9/13/21**  
6:30pm - 9:20pm  
**Online** Rev 2

**Not Permitted in Class**

Be sure to have all cellphones **OFF**  
*(unless used as calculator...)*  
**Although NOT required**  
**please turn on your cameras**

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**21 Fall Class Roster : 150-01 (Mon 6:30pm)**

**MATH150 – 01 Algebra & Trigonometry**

Last Name	First Name	Call Me	Time Zone
Garavelo	Naihra	Naihra	ET
Lin	Fanghao	Fanghao	ET
Nguyen	Khanh	Luci	ET
Powers	Tony	Tony	ET
Rakicevic-More	Alek	Alek	ET
Ramirez	Guillermo	Xavier	"- 2"
Richardson	Janie	Janie	ET
Wang	Ke Wei	Ke Wei	ET
Zawadski	Ela	Ela	ET
Zhang	Huiying	Hayley	"+12"

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**Instructor Contact Information**

**Dr. Charles Rubenstein** <crubent@pratt.edu>  
Professor of Engineering & Information Science  
**Brooklyn Campus Faculty Office: ARC G-49**

**Fall 2021 VIRTUAL Office hours ONLY**  
Thursdays: **10:00am - 1:00pm Via Zoom**  
*To make your appointment*  
*Send me an email at least one day in advance:*  
**crubent@pratt.edu**

**Subject line: 150 Office Hour**

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**Zoom Office Hours Info**

You **MUST** email me, not later than 12:00Noon on Tuesday, with preferred meeting time(s) to arrange for a fifteen (15) minute, one-on-one Zoom session during my Thursday Office Hours:

**Thursday Office Hour Zoom Meetings**  
**10:00am – 1:00pm**  
<https://pratt.zoom.us/j/5691762059?pwd=SnpsEEZnNnVUe8vE8vEzXENvUT09>  
**Meeting ID: 569 176 2059**  
**Passcode: Office**

Challenges? email me at **crubent@pratt.edu** !  
with the subject: **150 or Math**

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**Draft Schedule: Math 150 – Fall 2021 – Remote Learning**

Monday	Notes
30-Aug	1. Introduction: Numbers, Arithmetic Operations, Fractions
6-Sep	<i>Pratt Holiday - NO CLASSES – Labor Day</i>
13-Sep	2. Manipulation of Algebraic Expressions (H/Q1)
20-Sep	3. Solving Linear and Quadratic Equations of One Variable (H/Q2)
27-Sep	4. Solving Equations of Two Variables (H/Q3)
4-Oct	5. Creating Equations – Polynomials (H/Q4); <i>Exam #1 Sunday 10/3; 9am</i>
11-Oct	6. Polynomial Functions, continued (H/Q5); <i>Exam #1 Review</i>
18-Oct	7. Functions, Graphing, Exponents and Logarithms (H/Q6)
25-Oct	8. Trigonometric Functions, Pythagorean Theorem (H/Q7)
1-Nov	9. Applications of Trigonometry (H/Q8)
8-Nov	10. Analytic Trigonometry, Identities, Graphing (H/Q9) <i>Exam #2 Sunday 11/7 9am</i>
15-Nov	11. Areas and Volumes of Geometric Solids (H/Q10) <i>Exam #2 Review</i>
22-Nov	12. Systems of Equations and Inequalities
29-Nov	13. Series and Sequences, Review topics
6-Dec	14. Final Examination (3-hour) <i>Emailed Sunday 12/5 @ 9am due by 2:00pm</i>

**NOTE: Take home exams account for the 15<sup>th</sup> class session;**  
**Exams emailed Sunday before date noted by 9:00am – due back by 1:00pm**

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### This Class Session

- **Due:** Textbook readings
- **Lecture:** Manipulation of Algebraic Expressions
- **Due & Review:** Homework Set #01 by 12:00Noon ET  
(Quiz #01 based on Homework Submission...)

#### In class – Session 3:

- **Due** Textbook readings
- **Due & Review:** Homework Set #02 / Quiz #02
- **Lecture:** Solving Linear and Quadratic Equations of One Variable
- **Review Homework #01 in class**

#### In class – Session 4:

- **Due & Review:** Homework Set #03 / Quiz #03
- **Lecture:** Solving Equations of Two Variables
- **Review Homework #02 in class**

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### Submitting Your Assignments

**Homework Set #XX** are due to me, via email, not later than 12:00pm Noon ET the day due.

The filename for homework submissions is:

*lastname\_hwkXX.doc or  
lastname\_hwkXX.docx or  
lastname\_hwkXX.pdf*

The filename for Exams is:

*lastname\_exam1.doc (etc.)*

**NOTE:** *If you are creating hand-written pages as a picture, please open a word processing file (e.g., doc, docx, rtf) and then drag your jpg (etc.) file into the word processing file, oriented vertically, save it with the filename above and then email it to me.*

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### Class Session Archives

[www.CharlesRubenstein.com/150](http://www.CharlesRubenstein.com/150)

[/21fa02.pdf](#) (this slide set)\*

[/21fa02\\_h.pdf](#) (slide set as handouts)\*

\*Available by Wednesday evenings...

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### Math 150 – Chapter Topics

1. The Foundations of Algebra
2. Equations and Inequalities
3. Functions
4. Polynomial Functions
5. Rational Functions and Conic Sections
6. Exponential and Logarithmic Functions
7. The Trigonometric Functions
8. Analytic Trigonometry
9. Applications of Trigonometry
10. Systems of Equations and Inequalities
11. Matrices, Linear Systems, and Determinants
12. Topics in Algebra

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### Nomenclature

In today's class we will discuss:

- Algebraic Expression
- Polynomial Expression
- Argument
- Equation
- Inequality
- Radical Sign
- Absolute Value
- Magnitude

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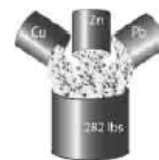
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### Chapter 1 - Page 12, Problem 60

Revisiting lasy week's problem...

*paying attention to rounding errors!*

60. An alloy is  $\frac{3}{8}$  copper,  $\frac{5}{12}$  zinc, and the balance lead. How much lead is there in 282 pounds of alloy?



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### Ch1, Pg12, Problem 60 - Ans

60. An alloy is  $\frac{3}{8}$  copper,  $\frac{5}{12}$  zinc, and the balance lead. How much lead is there in 282 pounds of alloy?


Equation is:  $(\frac{3}{8} + \frac{5}{12}) + x = 1$

a.  $\frac{3}{8} \cdot 3 \rightarrow \frac{9}{24}$  and  $\frac{5}{12} \cdot 2 \rightarrow \frac{10}{24}$   
 thus,  $\frac{19}{24} + x = 1$  and  $x_{\text{lead}} = \frac{5}{24} = 0.208^*$

b.  $282 \text{ lbs} \cdot 0.208 = 58.656 \text{ lbs}$  lead in the alloy

**Final Answer: 58.656 pounds**

*\*Using a calculator to find decimals and not using conversion to the lowest common denominator:  $(0.375 + 0.417) = 0.792$  and thus, again  $x_{\text{lead}} = 0.208$*



**BUT is this the correct answer? See next slide...**

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### Answers to 'n' decimal places

$x_{\text{lead}} = \frac{5}{24}$  Using  $\frac{5}{24} = 0.208333333333 \dots$

We get  $282 \text{ lbs} \cdot 0.208333333333 \dots = \mathbf{58.75 \text{ lbs}}$

Which is what we would get using fractions and NOT a calculator:  $(\frac{5}{24}) 282 = 235/4 = \mathbf{58 \frac{3}{4} \text{ lbs.}}$

Using  $x_{\text{lead}} = 0.208$  we get  $58.656 \text{ pounds}$ , and using  $x_{\text{lead}} = 0.21$  we get  $59.22 \text{ pounds!}$

**THEREFORE:**

*As both the 3 decimal place and 2 decimal place answers are approximations, unless told to, DON'T ROUND OFF YOUR NUMBERS!*

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# Questions?

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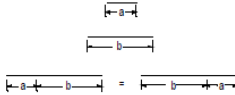
## Chapter 1 – Part 2

### The Foundations of Algebra

- 1.3 Algebraic Expressions & Polynomials
- 1.4 Factoring
- 1.5 Rational Expressions
- 1.6 Integer Exponents *(review on your own)*
- 1.7 Rational Exponents and Radicals

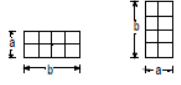
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### “Laws” - 1



$a + b = b + a$

**Commutative Law of Addition:**  
 $a + b = b + a$  (the order doesn't matter)




$ab = ba$

**Commutative Law of Multiplication:**  
 $ab = ba$  (the order doesn't matter)

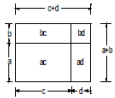
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### “Laws” - 2

**Associative Law of Addition:**  
 $a + (b+c) = (a+b) + c$



$(a+b)+c = ac + ad$



$(a+b)+c = ac + bc + ad + cd$

**Associative Law of Multiplication:**  
 $a(b+c) = ab + ac$   
*Division does not commute:  $a/b \dots b/a$  unless  $b = a$ .*

**Associative Law of Division:**  
 $(a+b)/c = (a/c) + (b/c)$  but  $a / (b+c) \dots (a/b) + (a/c)$

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### Positive and '0' Exponents

$a^2$  means  $a \cdot a$   
 $a^3$  means  $a \cdot a \cdot a$ , and so forth.  
 The **superscripts** 2 and 3 are known as **exponents**.

If we multiply  $a^2$  by  $a^3$  we have  
 $a^2 \cdot a^3 = (a \cdot a) \times (a \cdot a \cdot a) = a \cdot a \cdot a \cdot a \cdot a = a^5$   
**when a number raised to an exponent is multiplied by the same number raised to a different exponent, in the product the exponents add.**

**Note that  $a^1 = a$**

**What about ZERO exponents:**  
 Using this rule,  $a^n \cdot a^0 = a^{n+0} = a^n$ . Therefore  **$a^0 = 1$** .  
 Any number raised to the zeroth power is equal to one ("unity").

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### Negative and Fractional Exponents

**Negative exponents:**  
**using the rule for multiplication;**  $a^n \cdot a^{-n} = a^0 = 1$   
 For this equation to be true, we see that  $a^{-n} = 1/a^n$

**Fractional exponents:**  
**The multiplication rule gives us;**  $a^{1/2} \cdot a^{1/2} = a$   
 Since  $a^{1/2}$  times itself is equal to  $a$ ,  
 we see that  $a^{1/2} = \sqrt{a}$  = the square root of  $a$

Likewise,  $a^{1/3}$  is the cube root of  $a$ , etc.

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### Fractional Exponent Problems

Lets try to understand an expression such as  $3.3^{1.48}$   
 Breaking it into its exponents means  
 $(3.3^1)(3.3^{0.4})(3.3^{0.08})$  which can be expressed as "10's":  
 $(3.3)^1 (3.3^{1/10})^4 (3.3^{1/100})^8$

Note however, that we do not have to break it down this way to evaluate it; we can enter  $3.3^{1.48}$  directly with a calculator  
 to find  $3.3^{1.48} = ?$

**$3 \cdot 3 \wedge 1.48$  <enter>**  
**Results in: 5.853297943**

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### Graphing Calculator Alert

**Calculator Alert**

Your calculator evaluates exponents using a special key, which may be labeled  $\boxed{x^y}$ ,  $\boxed{y^x}$ , or  $\boxed{\wedge}$ .

**Example:**  $(1 + 2) \boxed{x^y} 3 = 0.125$   
 or  $(1 + 2) \boxed{y^x} 3 = 0.125$   
 or  $(1 + 2) \boxed{\wedge} 3 = 0.125$

WARNING  
 Note the difference between  
 $(-3)^2 = (-3)(-3) = 9$   
 and  
 $-3^2 = -(3 \cdot 3) = -9$

We will use  $\boxed{x^y}$  or  $\boxed{\wedge}$  to indicate the exponentiation key in this text.  
 In addition to the exponentiation key, your calculator probably has a special key labeled  $\boxed{x^{-1}}$ .

**Examples:**  $(-3) \boxed{x^{-1}} = 9$   
 $-3 \boxed{x^{-1}} = -9$

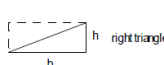
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Questions?


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### Geometry Review - 1

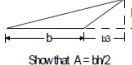
**Area of a triangle= 1/2 (base x height) A = (b x h)/2**  
 The triangle's apex doesn't have to lie above the base



right triangle  
A = bh/2 = half the rectangle



A = b1 h/2 + b2 h/2  
= (b1 + b2) h/2 = bh/2



Show that A = bh/2

A formula can be derived for the **area** of an arbitrary triangle in terms of the lengths of the three sides x, y, and z:

$$A = \frac{1}{4} \sqrt{2x^2y^2 + 2y^2z^2 + 2z^2x^2 - x^4 - y^4 - z^4}$$

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### Geometry Review - 2

**Circles and Spheres**

**Circumference of a circle:**  
 $C = 2 \pi r$  (contains radius length to the 1st power)

**Area of a circle:**  
 $A = \pi r^2$  (contains radius length to the 2nd power)

**Area of a sphere:**  
 $A = 4\pi r^2$  (contains radius length to the 2nd power)

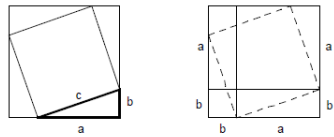
**Volume of a sphere:**  
 $V = (4/3) \cdot \pi r^3$  (contains radius length to the 3rd power)

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### Perfect Right Triangles - 1

#### The Pythagorean Theorem

*The sum of the squares of the sides equals the square of the hypotenuse...*



$c^2 + 2ab = a^2 + b^2 + 2ab$

Subtract 2ab from each side of the equation:

$a^2 + b^2 = c^2$

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### Perfect Right Triangles - 2

There is also an algorithm (recipe) to find perfect right triangles:  
 Pick any two integers  $i$  and  $j$ , where  $i > j$ .  
 The sides of the triangle will be  $i^2 + j^2$ ,  $i^2 - j^2$ , and  $2ij$

**Example:**  $i=2, j=1$ . Then the hypotenuse =  $i^2 + j^2 = 5$ ,  
 and the two sides are:  $i^2 - j^2 = 3$ , and  $2ij = 4$   
 The simplest perfect right triangle has sides 3, 4, and 5.  
 (Note that  $3^2 + 4^2 = 5^2$ )

Two other perfect right triangles are 5, 12, 13 and 8, 15, 17.  
 (check them)

**Problem to work out on your own:**  
 Use algebra to verify this recipe, that is, show that  
 $(i^2 - j^2)^2 + (2ij)^2 = (i^2 + j^2)^2$

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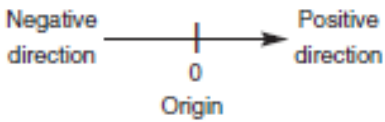
## Chapter 1.2

### The Real Number Line

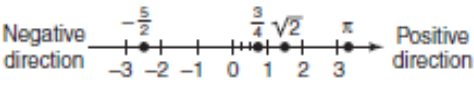
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### The Real Number Line

- Concept



- Example



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### Inequalities

Symbol	Meaning
$<$	Less than
$>$	Greater than
$\leq$	Less than or equal to
$\geq$	Greater than or equal to

Algebraic Expression	Meaning	Equivalent Statement	Geometric Statement
$a > 0$	$a$ is greater than 0.	$a$ is positive.	$a$ lies to the right of the origin.
$a < 0$	$a$ is less than 0.	$a$ is negative.	$a$ lies to the left of the origin.
$a > b$	$a$ is greater than $b$ .	$a - b$ is positive.	$a$ lies to the right of $b$ .
$a < b$	$a$ is less than $b$ .	$a - b$ is negative.	$a$ lies to the left of $b$ .
$a \geq b$	$a$ is greater than or equal to $b$ .	$a - b$ is positive or zero.	$a$ lies to the right of $b$ or coincides with $b$ .
$a \leq b$	$a$ is less than or equal to $b$ .	$a - b$ is negative or zero.	$a$ lies to the left of $b$ or coincides with $b$ .

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### Properties of Inequalities

Example	Algebraic Expression	Property
Either $2 < 3$ , $2 > 3$ , or $2 = 3$ . Since $2 < 3$ and $3 < 5$ , then $2 < 5$ . Since $2 < 5$ , then $2 + 4 < 5 + 4$ or $6 < 9$ .	Either $a < b$ , $a > b$ , or $a = b$ . If $a < b$ and $b < c$ then $a < c$ . If $a < b$ then $a + c < b + c$ .	Trichotomy property Transitive property The sense of an inequality is preserved if any constant is added to both sides.
Since $2 < 3$ and $4 > 0$ , then $2(4) < 3(4)$ or $8 < 12$ .	If $a < b$ and $c > 0$ , then $ac < bc$ .	The sense of an inequality is preserved if it is multiplied by a positive constant.
Since $2 < 3$ and $-4 < 0$ , then $2(-4) > 3(-4)$ or $-8 > -12$ .	If $a < b$ and $c < 0$ , then $ac > bc$ .	The sense of an inequality is reversed if it is multiplied by a negative constant.

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### Absolute Values

We use the definition of the Absolute Value:

$$|a| = \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a < 0 \end{cases}$$

$|-4| + |+4| = 4 + 4 = 8$   
*(This is NOT a subtraction!)*

To find the distance on a line from **-4 to +4**:

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### Graphing Calculator Alert

**Graphing Calculator Alert**

Your calculator may have an absolute value key, usually labeled **ABS**. If you have a graphing calculator, it is important to use parentheses when you use this key.

**Examples:**

- $ABS(5 - 2)$
- $ABS(2 - 5)$
- $ABS(3 - 5) - ABS(8 - 6)$
- $ABS(4 - 7) + (-6)$

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### Basic Properties of Absolute Value

Example	Algebraic Expression	Property
$ -2  \geq 0$ $ 3  =  -3  = 3$	$ a  \geq 0$ $ a  =  -a $	Absolute value is always nonnegative. The absolute values of a number and its negative are the same.
$ 2 - 5  =  -3  = 3$ $ 5 - 2  =  3  = 3$	$ a - b  =  b - a $	The absolute value of the difference of two numbers is always the same, irrespective of the order of subtraction.
$ (-2)(3)  =  -2  3  = 6$	$ ab  =  a  b $	The absolute value of a product is the product of the absolute values.

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## Chapter 1.3 Algebraic Expressions and Polynomials

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### Algebraic Expressions

“You invest  $p$  dollars at **6% simple annual interest** for 1 year.  
How much do you now have?”

- Variables ( $p$ )
- Constants ( $0.06$ )
- Algebraic Operations ( $+$ ,  $-$ ,  $/$ ,  $\times$ )
- Resultant (?)

**Resulting in...**

**$p + 0.06p = ?$  or  $p(1 + 0.06) = ?$**

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## Polynomials

Polynomials are equations of the form:

$$P = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \dots + a_1 x^1 + a_0$$

The  $P_{n-a}$  term of the equation =  $(a_{n-a} x^{n-a})$   
where:

Coefficients  $a_{n-1}$  are constant real numbers  
 $a_n \neq 0$ ,  $a = \text{real } \#$ , and  $n = \text{nonnegative integer}$

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## Degree of a Polynomial

*The degree of a polynomial is the exponent value of the highest monomial with a nonzero coefficient:*

$$x + y \quad \leftarrow \text{Degree 1}$$

$$xy \quad \leftarrow \text{Degree 2}$$

$$2x^2y + y^2 - 3xy + 1 \quad \leftarrow \text{Degree 3}$$

$$3x^4 + xy - y^3 \quad \leftarrow \text{Degree 4}$$

*Polynomials are equal if and only if all terms are equal*

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*Questions?*

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Chapter 1.4  
**Factoring**

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## Reducing Polynomials - Common Factors

$$x^2 + x = x(x + 1)$$

$$2xy + 2 = 2(xy + 1)$$

$$25x^3 - 10x^2 = 5x^2(5x - 2)$$

$$3x^4 + x^3 + xy = x(3x^3 + x^2 + y)$$

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## Factoring by Grouping

**GIVEN:**  $2ab + b + 2ac + c$

Grouping b, c:  $(2ab + b) + (2ac + c)$

Common factors b,c:

$$b(2a+1) + c(2a+1)$$

Common factors (2a+1):

$$(2a+1)(b+c) = \textit{final answer}$$

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### Factoring 2<sup>nd</sup> Degree Polynomials

**GIVEN:**  $x^2 - 7x + 10$

Constant is positive, middle term is negative; 2 @ -

Integer Pairs product =10: 1 & 10; =7: 2 & 5

Factoring:

$$(x-2)(x-5)$$

**GIVEN:**  $x^2 - 9$  (difference of squares)

$$(x+3)(x-3)$$

**In general;**  $a^2 - b^2 = (a+b)(a-b)$

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### Sum/Difference of Cubes

**GIVEN: Sum of cubes:**

$$a^3 + b^3 \\ = (a+b)(a^2 - ab + b^2)$$

**GIVEN: Difference of cubes:**

$$a^3 - b^3 \\ = (a-b)(a^2 + ab + b^2)$$

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### Irreducible Polynomials

*Prime or irreducible polynomials cannot be written as a product of two polynomials of positive degree...*

**Examples:**

$$x^2 + 1$$

$$x^2 + x + 1$$

*knowing these makes it easier to know when to 'stop' factoring...*

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### "MAGICAL" Factoring for Second-Degree Polynomials

*Factoring involves a certain amount of trial and error that can become frustrating, especially when the lead coefficient is not 1. We demonstrate the method for the polynomial  $4x^2 + 11x + 6 = ?$*  Eq. 1

(1) Using the lead coefficient of 4, write the pair of incomplete factors

$$(4x \pm ?)(4x \pm ?)$$
 Eq. 2

(2) Next, multiply the coefficient of  $x^2$  and the constant term in Equation (1) to produce  $4 \cdot 6 = 24$ . Now find two integers whose product is 24 and whose sum is 11, the coefficient of the middle term of (1). Since 8 and 3 work, and all signs are +, we can write

$$(4x + 8)(4x + 3)$$
 Eq. 3

Finally, within each parenthesis discard any common numerical factor. (Discarding a factor may only be performed in this "magical" type of factoring.) Thus  $(4x + 8)$  reduces to  $(x + 2)$  and we write

$$(x + 2)(4x + 3)$$
 Eq. 4

which is the factorization of  $4x^2 + 11x + 6$

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*Questions?*

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Chapter 1.5  
Rational Expressions

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### Rational Expressions

The reciprocal of  $(b/a)$  equals  $(a/b)$

Multiplication of rational expressions

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$$

The Rule of 'One':

$$a/a = 1; [d/c / d/c] = 1$$

$$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{\frac{a}{b} \cdot \frac{d}{d}}{\frac{c}{d} \cdot \frac{d}{d}} = \frac{\frac{a}{b} \cdot \frac{d}{1}}{\frac{c}{1} \cdot \frac{d}{d}} = \frac{a}{b} \cdot \frac{d}{c}$$

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### Cancellation

Cancel the common **4** and **y**:

$$\frac{4xy}{4yz} = \frac{x}{z}$$

Cancel the  $(2x^2+3)$  term:

$$\frac{(2x^2+3)(4x^3+4)}{(7x^7+1)(2x^2+3)} = \frac{(4x^3+4)}{(7x^7+1)}$$

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### Add/Subtract Fractions

Multiplying by '1' (7/7, 3/3)

$$\frac{2}{3} + \frac{1}{7} = \frac{2}{3} \cdot \frac{7}{7} + \frac{1}{7} \cdot \frac{3}{3} = \frac{14}{21} + \frac{3}{21} = \frac{17}{21}$$

Multiplying by '1' (7/7,  $(b+6)/(b+6)$ )

$$\frac{2a+5}{b+6} + \frac{a-4}{7} = \frac{2a+5}{b+6} \cdot \frac{7}{7} + \frac{a-4}{7} \cdot \frac{b+6}{b+6} = \frac{7(2a+5) + (a-4)(b+6)}{7(b+6)}$$

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### Cross Multiplication

Multiply the left-hand numerator by the right-hand denominator and visa versa

$$\text{GIVEN: } \frac{x}{y} = \frac{a}{b}$$

This is the same as the cross-multiplied:

$$xb = ya$$

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Chapter 1.6  
**Integer Exponents**  
*review on your own...*

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Chapter 1.7  
**Rational Exponents and Radicals**

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### Properties of Powers & Roots

For  $a^n = b$  and  $a = b^{1/n}$  for  $n > 0$

Example	Property
$2^3 = 8$	Any power of a real number is a real number.
$8^{1/3} = 2$	The odd root of a real number is a real number.
$0^n = 0$	A positive power or root of zero is zero.
$4^2 = 16$	A positive number raised to an even power equals the negative of that number raised to the same even power.
$(16)^{1/2} = 4$	The principal root of a positive number is a positive number.
$(-4)^{1/2}$ is undefined in the real number system.	The even root of a negative number is not a real number.

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### Properties of Radicals

Example	Property
$\sqrt[3]{8^2} = (\sqrt[3]{8})^2 = 4$	$\sqrt[n]{b^m} = (\sqrt[n]{b})^m$
$\sqrt{4}\sqrt{9} = \sqrt{36} = 6$	$\sqrt{a}\sqrt{b} = \sqrt{ab}$
$\frac{\sqrt[3]{8}}{\sqrt[3]{27}} = \sqrt[3]{\frac{8}{27}} = \frac{2}{3}$	$\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$
$\sqrt[3]{(-2)^3} = -2$	$\sqrt[n]{a^n} = a$ if $n$ is odd
$\sqrt{(-2)^2} =  -2  = 2$	$\sqrt[n]{a^n} =  a $ if $n$ is even

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## In Class Problems

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### More Difficult Problems

*If you take out a mortgage for \$49,000 for 10 years at an interest rate of 5.25%, how much is the monthly payment?*

where  $m$  is the monthly payment,

$P$  is the original principal (49,000),

$I$  is the interest rate (0.0525),

and  $Y$  is the number of years (10)

$$m = \frac{P \frac{I}{12}}{1 - \left(1 + \frac{I}{12}\right)^{-12Y}}$$

*This problem is much more difficult than ones we will normally work with, but still within the scope of this course*

The answer is  $m = \$625.73$

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### Quick Problems

Ch. 1.1, Pg 12 (5<sup>th</sup> 11) #63.

*A woman's take-home pay is \$210.00 after deducting 18% withholding tax. What is her pay before the deduction?*

You have 5 minutes to solve this

18% is deducted from 100% pay leaving  $100\% - 18\% = 82\%$  for net pay. Thus, 210 is 82% of her gross earnings:

$$210 = 0.82x$$

$x = 210/0.82 = \$256.10$  (note the '\$' in the answer...)

*This is the question type where Net = (% Gross).*

*To find the amount of tax: Gross - Net = Tax*

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### Quick Problems

Ch. 1.1, Pg 12 (5<sup>th</sup> 11) #66.

*Eric starts at a certain time driving his car from New York to Philadelphia going 50 mph. Sixty minutes later, Steve leaves in his car en route from Philadelphia to New York going 40 mph.*

*When the two cars meet, which one is nearer to New York?*

You have 1 minute to solve this

*Although we could put a timeline to try to show where the vehicles are at time  $t=60$  minutes, etc., when the cars meet they are BOTH the same distance from both NY and Philly.*

***This was a trick question...***

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### Quick Problems

Ch. 1.3, Pg 29 (5<sup>th</sup> 28) #60. Perform the indicated operation:

$$5(2x - 3)^2$$

You have 3 minutes to solve this

$$\begin{aligned} &= 5(2x - 3)(2x - 3) \\ &= 5(4x^2 - 12x + 9) \\ &= (20x^2 - 60x + 45) \text{ final answer} \end{aligned}$$

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### Quick Problems

Ch. 1.3, Pg 29 (5<sup>th</sup> 28) #62. Perform the indicated operation:

$$(x - 1)(x + 2)(x + 3)$$

You have 3 minutes to solve this

$$\begin{aligned} &= (x^2 + x - 2)(x + 3) \\ &= x^2(x + 3) + x(x + 3) - 2(x + 3) \\ &= x^3 + 3x^2 + x^2 + 3x - 2x - 6 \\ &= x^3 + 4x^2 + x - 6 \text{ final answer} \end{aligned}$$

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### Quick Problems

Ch. 1.3, Pg 29 (5<sup>th</sup> 28) #63 An investor buys  $x$  shares of IBM stock at \$98 per share at Thursday's opening of the stock market. Later in the day, the investor sells  $y$  shares of AT&T stock at \$38 per share and  $z$  shares of TRW stock at \$20 per share. Write a polynomial that expresses the amount of money the buyer has invested at the end of the day.

You have 3 minutes to solve this

pays for IBM stock:  $-98x$   
 sells AT&T stock:  $+38y$   
 sells TRW stock:  $+20z$

$$\text{INVESTMENT} = -98x + 38y + 20z \text{ final answer}$$

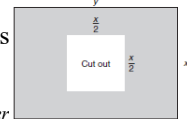
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### Quick Problems

Ch. 1.3, Pg 30 (5<sup>th</sup> 28) #64. An artist takes a rectangular piece of cardboard whose sides are  $x$  and  $y$  and cuts out a square of side  $x/2$  to obtain a mat for a painting, as shown in Figure 5. Write a polynomial giving the area of the mat.

You have 3 minutes to solve this



$$\begin{aligned} \text{Area}_{\text{mat}} &= \text{Area}_{\text{outer}} - \text{Area}_{\text{inner}} \\ &= xy - (x/2)(x/2) \\ &= xy - (x/2)^2 \\ &= xy - (x^2/4) \text{ final answer} \end{aligned}$$

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### Quick Problems

Ch. 1.3, Pg 30 (5<sup>th</sup> 28) #73. Perform the multiplication mentally:

$$(3x - 1)^2$$

You have 2 minutes to solve this

$$\begin{aligned} &= (3x - 1)(3x - 1) \\ &= 9x^2 - 3x - 3x + 1 \\ &= 9x^2 - 6x + 1 \text{ final answer} \end{aligned}$$

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### Quick Problems

Ch. 1.3, Pg 30 (5<sup>th</sup> 28) #74.

Perform the multiplication mentally:

$$(x + 2)(x - 2)$$

You have 2 minutes to solve this

$$\begin{aligned} (x + 2)(x - 2) &= x(x - 2) + 2(x - 2) \\ &= x^2 - 2x + 2x - 4 \\ &= x^2 - 4 \text{ (difference of squares) final ans.} \end{aligned}$$

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## Homework Set #02\*

**Section 1.3** (Expressions and Polynomials)

Page 28: Problems 4, 27, 42, 44, 49, 50, 58, 82

**Section 1.4** (Factoring)

Page 38: Problems 2, 9, 11, 12, 13, 32, 40

**Section 1.5** (Rational Expressions)

Page 47: Problems 1, 2, 7, 8, 25, 32, 51, 52

**Section 1.7** (Rational Exponents and Radicals)

Page 67: Problems 1, 5, 6, 19, 20

*\*All homework assignments are on the class website*

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## Topics in Session 3

**Ch. 1 The Foundations of Algebra**

**1.8 Complex Numbers**

**Chapter 1 Review**

**Ch. 2 Equations and Inequalities**

**2.1 Linear Equations in One Unknown**

**2.2 Applications: From Words to Algebra**

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## In Class Session 03

- **Due:** Textbook readings
- **Due & Review:** Homework Set #02 / Quiz #02
- **Lecture:** Solving Linear and Quadratic Equations of One Variable
- **Review Homework #01**

### **In class – Session 4**

- **Due:** Textbook readings
- **Due & Review:** Homework Set #03 / Quiz #03
- **Lecture:** Solving Equations of Two Variables
- **Review Homework #02**

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Any Questions?  
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**End**

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