


Pratt



**Math 150 – Fall 2021
Algebra & Trigonometry**
Charles Rubenstein, Ph. D.
Professor of Information Science

Session 4: Monday 9/27/21
6:30pm - 9:20pm
Online – Revision 2

Not Permitted in Class



Be sure to have all cellphones **OFF**
(unless used as calculator...)
Although NOT required
please turn on your cameras

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21 Fall Class Roster : 150-01 (Mon 6:30pm)

MATH150 – 01 Algebra & Trigonometry

Last Name	First Name	Call Me	Time Zone
Garavelo	Naihra	Naihra	ET
Lin	Fanghao	Fanghao	ET
Nguyen	Khanh	Luci	ET
Powers	Tony	Tony	ET
Rakicevic-More	Alek	Alek	ET
Ramirez	Guillermo	Xavier	"- 2"
Richardson	Janie	Janie	ET
Wang	Ke Wei	Ke Wei	ET
Zawadski	Ela	Ela	ET
Zhang	Huiying	Hayley	"*12"

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Instructor Contact Information

Dr. Charles Rubenstein <crubens@pratt.edu>
Professor of Engineering & Information Science
Brooklyn Campus Faculty Office: ARC G-49

Fall 2021 VIRTUAL Office hours ONLY
Thursdays: 10:00am - 1:00pm *Via Zoom*
Meeting ID: 569 176 2059
Passcode: Office

To make your appointment, please send me an email at least one day in advance:
crubens@pratt.edu
Subject line: 150 Office Hour

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US Citizens – Student Alert!

To vote in November you must register. Contact dkahn@nypirg.org



Register & VOTE!

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Draft Schedule: Math 150 – Fall 2021 – Remote Learning

Monday	Notes
30-Aug	1. Introduction: Numbers, Arithmetic Operations, Fractions
6-Sep	<i>Pratt Holiday - NO CLASSES – Labor Day</i>
13-Sep	2. Manipulation of Algebraic Expressions (H/Q1)
20-Sep	3. Solving Linear and Quadratic Equations of One Variable (H/Q2)
27-Sep	4. Solving Equations of Two Variables (H/Q3)
4-Oct	5. Creating Equations – Polynomials (H/Q4); <i>Exam #1 Sunday 10/3; 9am</i>
11-Oct	6. Polynomial Functions, continued (H/Q5); <i>Exam #1 Review</i>
18-Oct	7. Functions, Graphing, Exponents and Logarithms (H/Q6)
25-Oct	8. Trigonometric Functions, Pythagorean Theorem (H/Q7)
1-Nov	9. Applications of Trigonometry (H/Q8)
8-Nov	10. Analytic Trigonometry, Identities, Graphing (H/Q9) <i>Exam #2 Sunday 11/7 9am</i>
15-Nov	11. Areas and Volumes of Geometric Solids (H/Q10) <i>Exam #2 Review</i>
22-Nov	12. Systems of Equations and Inequalities
29-Nov	13. Series and Sequences, Review topics
6-Dec	14. Final Examination (3-hour) <i>Emailed Sunday 12/5 @ 9am due by 2:00pm</i>

*NOTE: Take home exams account for the 15th class session;
Exams emailed Sunday before date noted by 9:00am – due back by 1:00pm*

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About Your Final Grade

Homework (30%)

For the first **ten (10)** Homework assignments I will select three (3) problems from each assignment to grade. 1% per correct answer, or **3% per homework**

Homework must be emailed to me by 12:00Noon ET on day of class as a doc, rtf, pdf or other file – not as a photo/jpg. The filename **MUST** be **lastname_hwk##.docx** (or *doc, pdf, etc.*)

Exams (70%)

There will be two (2) one-hour exams worth **20% each**
 There will be a two-hour **FINAL** exam worth **30%**

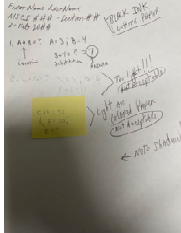
Exams will be emailed to you as noted in the schedule on a Sunday not later than 9:00am ET and must be returned to me not later than 1:00pm ET the same day.

There will be NO make up 'quizzes' or exams

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Creating a 'Doc' file for a JPG

- 1. Take a photo of your work**
 – **BE SURE IT IS IN BLACK INK ON WHITE PAPER** – make sure your work is clear and readable. If I can not read it: too light, too sloppy, etc., I will not be able to grade your work!
- 2. Open a doc, docx, or rtf file.**
- 3. Insert your name, class/section, and date in the file.** (Note: if scanning, make sure this info is on the page and output a pdf.)
- 4. Drag your picture file into the document file and make sure it is oriented properly.**
- 5. Save the file:** The filename **MUST** be styled like **lastname_hwk##.docx** (or rtf, doc, pdf, etc.)
lastname_exam#.docx (or rtf, doc, pdf, etc.)



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In This Class Session

- **Due Textbook readings**
- **Due & Review: Homework Set #03 / Quiz#03**
- **Lecture: Equations of Two Variables**
- **Review: Homework Set #02 in class**

EXAM 1 – Sunday 3 October

Exam will be emailed to you not later than 9:00am ET on 10/3/21 and must be returned to me not later than 1:00pm ET the same day.

In class – Session 5:

- **Due: Homework Set #04 / Quiz#04**
- **Lecture: Creating Equations - Polynomials**
- **In Class Review: Exam 1; Homework Set #03**

About the CM/FM Seminar Series...
Wednesday 3 November
 Via Zoom – details to follow

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Class Session Archives

www.CharlesRubenstein.com/150

21fa04.pdf (this slide set)*
21fa04_h.pdf (slide set as handouts)*

**Available by Thursday evenings...*

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Math 150 – Chapter Topics

- 1. The Foundations of Algebra**
- 2. Equations and Inequalities**
- 3. Functions**
- 4. Polynomial Functions**
- 5. Rational Functions and Conic Sections**
- 6. Exponential and Logarithmic Functions**
- 7. The Trigonometric Functions**
- 8. Analytic Trigonometry**
- 9. Applications of Trigonometry**
- 10. Systems of Equations and Inequalities**
- 11. Matrices, Linear Systems, and Determinants**
- 12. Topics in Algebra**

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Questions?

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Topics in Session 4

Ch. 2 Equations and Inequalities
 2.3 The Quadratic Equation
 2.4 Applications of Quadratic Equations
 2.5 Linear and Quadratic Inequalities
 2.6 Absolute Value in Equations and Inequalities

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Homework #02
Selected
Review Problems

Average Quiz Grade: 2%

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Problems – Homework #2

Ch. 1.4, Pg 38
 #2. Factor completely... $\frac{1}{4}x + \frac{3}{4}y$

$$\frac{1}{4}x + \frac{3}{4}y =$$

$$\frac{1}{4}(x + 3y) \quad \text{final ans.}$$

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Problems – Homework #2

Ch. 1.5, Pg 47 #1, 2, 7, 8, 32, 52
 Prob. 1. $\frac{x+4}{x^2-16}$

Since $(x^2 - 16) = (x+4)(x-4)$
 we can write

$$\frac{\cancel{(x+4)}}{\cancel{(x+4)}(x-4)} = \frac{1}{(x-4)} \quad \text{final ans.}$$

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Problems – Homework #2

Ch. 1.5, Pg 47 #1, 2, 7, 8, 32, 52
 Prob. 2. $\frac{y^2 - 25}{y+5}$

Since $(y^2 - 25) = (y+5)(y-5)$ we can write
 which simplifies $\frac{\cancel{(y+5)}(y-5)}{\cancel{(y+5)}}$

$$= (y-5) \quad \text{final ans.}$$

QUIZ PROBLEM

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Problems – Homework #2

Ch. 1.5, Pg 47 #1, 2, 7, 8, 32, 52
 Prob. 7. $\frac{2}{3x-6} \div \frac{3}{2x-4}$

$$= \frac{2}{3x-6} = \frac{2}{3(x-2)} \cdot \frac{(6)(x-2)}{(6)(x-2)}$$

$$= \frac{2}{3} \cdot \frac{6}{2(x-2)}$$

$$= \frac{4}{9}, x \neq 2 \quad \text{final ans.}$$

(Note: $x \neq 2$ since $(x-2)$ would then be zero, undefined!)

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Problems – Homework #2

Ch. 1.5, Pg 47 #1, 2, 7, 8, 32, 52

Prob. 8.

$$\frac{5x + 15}{8} \div \frac{3x + 9}{4} = \frac{5(x+3)}{3(x+3)} \cdot \frac{4}{8}$$

$$= \frac{5(x+3)}{3(x+3)} \cdot \frac{(8)}{(8)} = \frac{5(x+3)}{6(x+3)}$$

$$= \frac{5}{6}, x \neq -3 \quad \text{final ans.}$$

(Note: $x \neq -3$ since $(x+3)$ would then be zero, undefined!)

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Problems – Homework #2

Ch. 1.5, Pg 47 **Prob. 32.**

$$\frac{x}{x^2 - 4} + \frac{2}{4 - x^2} = \frac{x}{x^2 - 4} + \frac{2}{(-1)(x^2 - 4)}$$

$$= \frac{x}{x^2 - 4} - \frac{2}{x^2 - 4}$$

$$= \frac{x - 2}{x^2 - 4}$$

$$= \frac{x - 2}{(x + 2)(x - 2)}$$

$$= \frac{1}{x + 2} \quad \text{final ans.}$$

QUIZ PROBLEM

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Problems – Homework #2

Ch. 1.5, Pg 47 #1, 2, 7, 8, 32, 52

Prob. 52.

$$\frac{x - \frac{1}{x}}{2 + \frac{1}{x}} = \frac{x - \frac{1}{x}}{2 + \frac{1}{x}} \cdot \frac{(x)}{(x)}$$

$$= \frac{x^2 - 1}{2x + 1} \quad \text{final ans.}$$

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Problems – Homework #2

Ch. 1.7, Pg 67 # 5, 6

Prob. 5.

$$\frac{2x^{1/3}}{x^{-3/4}} = 2x^{\frac{1}{3} - (-\frac{3}{4})} = 2x^{\frac{4}{12} + \frac{9}{12}} = 2x^{\frac{13}{12}} \quad \text{final ans.}$$

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Problems – Homework #2

Ch. 1.7, Pg 67 # 5, 6

Prob. 6.

$$\frac{y^{-2/3}}{y^{1/5}} = y^{-\frac{2}{3} - \frac{1}{5}}$$

$$= y^{-\frac{10}{15} - \frac{3}{15}}$$

$$= y^{-\frac{13}{15}} = \frac{1}{y^{13/15}} \quad \text{final ans.}$$

QUIZ PROBLEM

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Questions?

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In Chapter 2

Equations and Inequalities

- 2.1 Linear Equations in One Unknown
- 2.2 Applications: From Words to Algebra
- 2.3 The Quadratic Equation
- 2.4 Applications of Quadratic Equations
- 2.5 Linear and Quadratic Inequalities
- 2.6 Absolute Value in Equations and Inequalities

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Chapter 2

2.3 The Quadratic Equation

$$ax^2 + bx + c = 0$$

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REVIEW: Solving by Factoring

We can factor the left hand side of the Quadratic Equation

$$ax^2 + bx + c = 0$$

into two linear factors to solve the equation.

So for the quadratic, $x^2 + 5x + 6 = 0$

Factoring into: $(x + 2)(x + 3) = 0$

This is only true for $(x + 2) = 0$ or $(x + 3) = 0$

Thus we have two possible results:

$$x = -2 \text{ AND } x = -3$$

that will satisfy the equation

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Quick Problems

Ch. 2, Pg 109 #a

Solve the equation: $4x^2 - x = 0$

You have 3 minutes to solve this

$$4x^2 - x = 0 \rightarrow x(4x - 1) = 0$$

Resist the temptation to solve by dividing by x as the solution requires TWO values of x :

$$x = 0^* \text{ and } x = \frac{1}{4} \text{ final answer}$$

*NOTE: $x = 0$ may not always be realistic, but might be possible

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Quick Problems

Now solve the equation: $4x^2 - 4x = 0$

You have 2 minutes to solve this

$$4x^2 - 4x = 0 \rightarrow 4x(x - 1) = 0$$

Resisting the temptation to solve by dividing by x as the solution requires TWO values of x :

$$4x = 0 \rightarrow x = 0$$

$$x - 1 = 0 \rightarrow x = 1$$

$$x = 0^* \text{ and } x = 1 \text{ final answer}$$

*NOTE: $x = 0$ may not always be realistic, but might be possible

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Special Cases: $a(x+h)^2 + c = 0$

Equations of the form $a(x+h)^2 + c = 0$

May be solved in a straightforward manner:

$$a(x+h)^2 + c = 0$$

$$a(x+h)^2 = -c$$

$$(x+h)^2 = -c/a$$

$$(x+h) = \pm\sqrt{-c/a}$$

and thus

$$x = -h \pm\sqrt{-c/a}$$

NOTE: Only valid for CM/FM if c or a are negative making $-c/a$ positive...

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Special Case Example

Example 7; Pg 112

Where $2(x-1)^2 - 6 = 0$

We solve in a straightforward manner:

$$2(x-1)^2 = 6$$

$$(x-1)^2 = 3$$

$$(x-1) = \pm\sqrt{3}$$

and thus $x = 1 \pm\sqrt{3}$

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Quick Problems

Ch. 2, Pg 113 #b

Solve the equation: $(2x - 7)^2 - 5 = 0$

You have 3 minutes to solve this

$$(2x - 7)^2 - 5 = 0 \rightarrow (2x - 7)^2 = 5$$

$$(2x - 7) = \pm\sqrt{5}$$

$$2x = 7 \pm\sqrt{5}$$

$$x = \frac{7 \pm\sqrt{5}}{2} \quad \text{final answer}$$

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Completing the Square (optional)

ANY quadratic equation can be rewritten in the form

$$a(x+h)^2 + c = 0$$

By the technique of "completing the square"

If we start with: $x^2 + dx$ we are missing the constant h^2

such that we have: $x^2 + dx + h^2 = (x+h)^2$

Expanding the right side: $x^2 + dx + h^2 = x^2 + 2hx + h^2$

Therefore $dx = 2hx$

such that $h = d/2$

$$\text{and } h^2 = (d/2)^2$$

To complete the square add $h^2 = (d/2)^2$ to $x^2 + dx$

Yielding the equation: $x^2 + dx + (d/2)^2 = 0$

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Completing the Square Example

Example 10 Pg 114

Solve by completing the square: $2x^2 - 10x + 1 = 0$

In a straightforward manner:

$$2x^2 - 10x = -1 \quad \text{therefore} \quad 2(x^2 - 5x) = -1$$

$$x^2 - 5x = -\frac{1}{2} \quad \text{eq. 1}$$

$$h = d/2; h = -5/2 \rightarrow h^2 = (d/2)^2; h^2 = 25/4$$

Adding to both sides of equation 1: $x^2 - 5x + 25/4 = -\frac{1}{2} + 25/4$

$$(x - 5/2)^2 = 23/4 \quad \text{thus} \quad x - 5/2 = \pm(\sqrt{23}/4)$$

Such that $x = 5/2 \pm(\sqrt{23})/2$

and finally $x = \frac{1}{2}(5 \pm\sqrt{23})$

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The Quadratic Formula

Once we write the quadratic equation in the form

$$ax^2 + bx + c = 0 \quad \text{for } a \neq 0$$

We can solve the equation in the most general terms

To find the general expression for the two roots of the equation:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

NOTE!!! This is NOT reducible to:

$$x \neq -b \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

Please be careful if you use your calculator to solve these equations as it is easy to make entry errors...

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Quadratic Formula Example

Example 11 Pg 115

Solve: $2x^2 - 3x - 3 = 0$ using the quadratic formula

In a straightforward manner: $a = 2, b = -3, c = -3$

Substitute into the formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$\text{Such that} \quad x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(-3)}}{2(2)}$$

$$x = \frac{+3 \pm \sqrt{9 + 24}}{4}$$

$$x = \frac{(3 \pm \sqrt{33})}{4}$$

and finally $x = \frac{1}{4}(3 \pm \sqrt{33})$

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The Discriminant

The roots of the quadratic equation in the form
 $ax^2 + bx + c = 0$ for $a \neq 0$

may be solved using the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Where $b^2 - 4ac$ is called the **discriminant**:

When $b^2 - 4ac$ is positive \rightarrow roots are real numbers

When $b^2 - 4ac$ is zero \rightarrow there is a double root

When the discriminant result is a square, the roots are rational

When $b^2 - 4ac$ is negative \rightarrow roots are complex conjugate pairs

We do not use complex numbers in CM/FM...

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Determinant Examples

DO NOT Solve, express nature of the roots for the following equations:

$$2x^2 - 3x - 3 = 0$$

$b^2 - 4ac$ is $(3)^2 - 4(2)(-3) = 9 + 24 = +33 = \text{not a square}$

Roots = two real numbers

Pg 119 #a.

$$4x^2 - 20x + 25 = 0$$

$b^2 - 4ac$ is $(-20)^2 - 4(4)(25) = 400 - 400 = 0$

Roots = real double roots

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Determinant Examples

DO NOT Solve, express nature of the roots for the following equations:

Pg 119 #b

$$5x^2 - 6x = -2$$

$b^2 - 4ac$ is $(-6)^2 - 4(5)(+2) = 36 - 40 = -4 = \text{square}$

Roots = complex conjugate pair roots

Pg 119 #c

$$10x^2 = x + 2$$

$b^2 - 4ac$ is $(-1)^2 - 4(10)(-2) = 1 + 80 = +81 = \text{square}$

Roots = two real, rational roots

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Radical Equations

One form of equation that leads to the quadratic equation is the **Radical Equation** of the form:

$P = Q$ which is a subset of the solution set of

$$P^n = Q^n \text{ for } n \text{ a natural number}$$

Solutions of these equations **MUST** be checked to confirm that they satisfy the original equations...

If one solution is **NOT** a solution of the original equation it is called an **Extraneous Solution**.

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Questions?

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Chapter 2

2.4 Applications of Quadratic Equations

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Word Problems

Word problems can also lead to solutions by the quadratic equation;

“The larger of two positive numbers exceeds the smaller by 2. If the sum of the squares of the two numbers is 74, find the two numbers.”

So: $x =$ larger number; $(x-2) =$ smaller number

Sum of the squares = 74 and thus the equation is:

$$(x)^2 + (x-2)^2 = 74$$

As shown in the text, $x = 7$ and $x = -5$

BUT, -5 is not a solution as $x-2 = -7$; $(7)^2 + (-7)^2 \neq 74$

Therefore we use $x = 7$, and the smaller # = 5

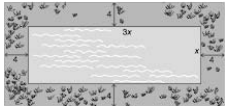
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Quick Problems

Ch. 2.4, Example 2. *“The length of a pool is 3 times its width, and the pool is surrounded by a grass walk 4 feet wide. If the total area covered and enclosed by the walk is 684 square feet, find the dimensions of the pool.”*

Defining Equation:

$A = l \times w$



$$(3x + 8)(x + 8) = 684$$

$$3x^2 + 32x + 64 = 684$$

$$3x^2 + 32x - 620 = 0$$

$$(3x + 62)(x - 10) = 0$$

x can't be negative; so we use $x = 10$

Pool is 10ft x 30ft ← ans

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Quick Problems

Ch. 2.4, Example 3. *“Working together, two cranes can unload a ship in 4 hours. The slower crane, working alone, requires 6 hours more than the faster crane to do the job. How long does it take each crane to do the job by itself?”*

Defining Equations:

$x =$ # hours/job for crane 1

$x + 6 =$ # hours/job for crane 2

$$4(1/x) + 4(1/(x+6)) = 1$$

$$4(x+6) + 4x = x^2 + 6x$$

$$0 = x^2 - 2x - 24$$

$$0 = (x+4)(x-6)$$

$x = -4$ rejected; $x = 6$

Fast crane does in 6 hours what slow crane does in 12 ← ans

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Questions?

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In Chapter 3

Functions

3.1 The Rectangular Coordinate System

3.2 Functions and Function Notation

3.3 Graphs of Functions

3.4 Linear Functions

3.5 The Algebra of Functions; Inverse Functions

3.6 Direct and Inverse Variation

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Chapter 3

3.1 The Rectangular Coordinate System

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Rectilinear Coordinates

The x-y Coordinate System

Where would you expect to find the following (x, y) points and Which Quadrant are they in?
 A: $(0,0)$ B: $(1, 1)$ C: $(1, -1)$ D: $(-1, 1)$ E: $(-1, -1)$
 F: $(10,0)$ G: $(0, 5)$ H: $(5, -5)$ I: $(-5, 5)$ J: $(-2, -2)$

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Rectilinear Coordinates

Locating points on the x-y Coordinate System

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The Distance Formula

The Distance between two points: $(-3, -1)$ and $(1, 2)$

Create a line parallel to the x axis from point $(-3, -1)$ and another line parallel to the y axis from point $(1, 2)$. In this case, the lines meet at $(1, -1)$. Line $d_1 = 3+1 = 4$ and line $d_2 = 2+1 = 3$. This figure is a right triangle. Therefore the hypotenuse 'd' is defined by: $d^2 = d_1^2 + d_2^2$ and $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
 Here we find $d = \sqrt{25} = 5$

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The General Distance Formula

In general the Distance between two points $P = (x_1, y_1)$ and $Q = (x_2, y_2)$ is found using the Pythagorean Theorem

$$d_1 = (x_2 - x_1) \text{ and } d_2 = (y_2 - y_1)$$

$$d^2 = d_1^2 + d_2^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

Therefore the Pythagorean hypotenuse 'd' is defined by:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

When this equation is correct, the figure is always a triangle...

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The Midpoint Formula

You can also find the midpoint M of any line segment...
 If we find the lengths of the lines: $d_1 = (x_2 - x_1)$ & $d_2 = (y_2 - y_1)$ all we need to do is divide these by '2' to find the average line length or midpoint.
 In general:

$$x_m = (x_2 - x_1) / 2 \text{ and similarly } y_m = (y_2 - y_1) / 2$$

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Questions?

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Chapter 3

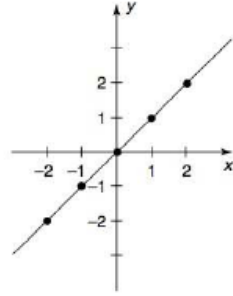
3.3 Graphs of Functions

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$f(x) = x$ Identity Function

$y = x$ (increasing)

x	y
-2	-2
-1	-1
0	0
1	1
2	2

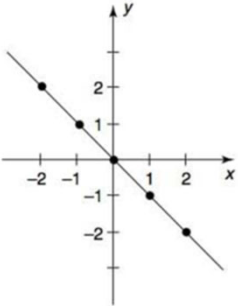


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$f(x) = -x$ Negation Function

$y = -x$ (decreasing)

x	y
-2	2
-1	1
0	0
1	-1
2	-2

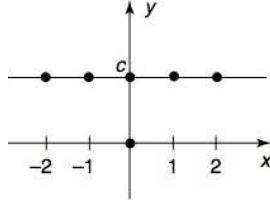


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$f(x) = c$ Constant Function

$y = c$ (constant)

x	y
-2	c
-1	c
0	c
1	c
2	c

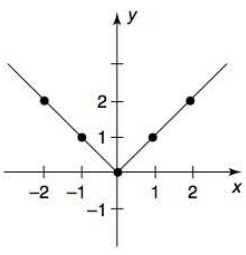


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$f(x) = |x|$ Absolute Value Function

$y = |x|$ (decreasing, then increasing)

x	y
-2	2
-1	1
0	0
1	1
2	2

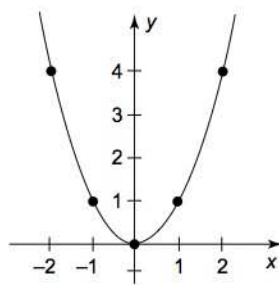


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$f(x) = x^2$ Squaring Function

$y = x^2$ (decreasing, then increasing)
= a Parabola

x	y
-2	4
-1	1
0	0
1	1
2	4



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Vertical Shifting of $f(x) = x^2 \pm c$
 $y = x^2 \pm c$
Parabolas

Graphs showing the parent function $y = x^2$ and its vertical shifts: $y = x^2 + 2$, $y = x^2 + 1$, $y = x^2$, $y = x^2 - 1$, and $y = x^2 - 2$.

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$f(x) = x^3$ Cubing Function
 $y = x^3$

x	y
-2	-8
-1	-1
0	0
1	1
2	8

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$f(x) = (x \pm c)^3$ Horizontal Shifts
 $y = (x \pm c)^3$ (cubic or cubing function)

Graphs showing the parent function $y = x^3$ and its horizontal shifts: $y = (x+2)^3$, $y = (x+1)^3$, $y = (x-1)^3$, and $y = (x-2)^3$.

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$f(x) = \sqrt{x}$ Square Root Function
 $y = \sqrt{x}$

x	y
0	0
1	1
4	2
9	3

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$f(x) = \pm x^2$ Reflections...

Graphs showing the parent function $y = x^2$ and its reflection $y = -x^2$.

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Shifting Graphs...

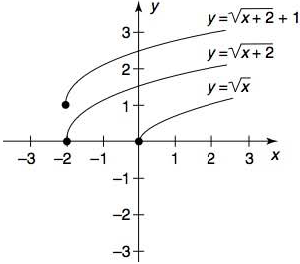
Form	Relationship to the Graph of $y = f(x)$, $p > 0$
$y = f(x) + p$	Shift $f(x)$ p units up.
$y = f(x) - p$	Shift $f(x)$ p units down.
$y = f(x - p)$	Shift $f(x)$ p units to the right.
$y = f(x + p)$	Shift $f(x)$ p units to the left.
$y = -f(x)$	Reflect $f(x)$ about the x -axis.
$y = f(-x)$	Reflect $f(x)$ about the y -axis.

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Quick Problem...

Sketch the graph of $y = \sqrt{x+2} + 1$

*This is the graph $y = \sqrt{x}$ shifted **two units to the left**, and then shifted **one unit up**.*



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Questions?

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Word Problem...

The commission earned by a door-to-door cosmetics salesperson is determined as shown in the following table.

Weekly Sales	Commission
Less than \$300	0.20s
\$300 or more, but less than \$400	60 + 0.40(s-300)
\$400 or more	100 + 0.60(s-400)

- Express the commission C as a function of sales s .
- Find the commission if the weekly sales are \$425.
- Sketch the graph of the function.

a. The function C can be described by three equations:

$$C(s) = \begin{cases} 0.20s & \text{if } 0 \leq s < 300 \\ 60 + 0.40(s - 300) & \text{if } 300 \leq s < 400 \\ 100 + 0.60(s - 400) & \text{if } s \geq 400 \end{cases}$$

b. When $s = 425$, we use the third equation:

$$C(425) = 100 + 0.60(425 - 400) = 100 + 0.60(25) = 115$$

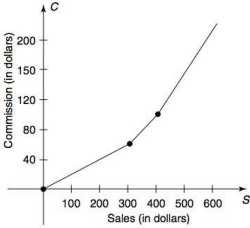
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Word Problem continued

The function C can be plotted using the three equations

$$C(s) = \begin{cases} 0.20s & \text{if } 0 \leq s < 300 \\ 60 + 0.40(s - 300) & \text{if } 300 \leq s < 400 \\ 100 + 0.60(s - 400) & \text{if } s \geq 400 \end{cases}$$

c. Sketch the graph of the function.
where $s=0, C=0$; when $s=300, C=60$; and when $s=425, C=115$:



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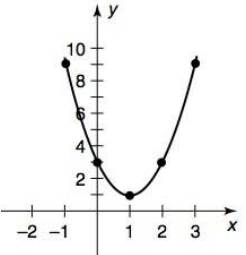
$f(x) = ax^2 + bx + c$ Polynomial Functions

$y = ax + b$ (linear function)

$y = ax^2 + bx + c$ (quadratic function) $a \neq 0$

For Example: $y = 2x^2 - 4x + 3$

x	y
-1	9
0	3
1	1
2	3
3	9



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Chapter 3

3.4 Linear Functions

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The Slope of a Line

The polynomial $f(x) = ax + b$ is a linear function and when graphed, will show a straight line...

We can readily show that if this line is not vertical or horizontal, moving from one point on the line to another can be calculated using the differences in the x's and y's in a ratio called the slope of the line "m" where:

$$m = (y - y_1) / (x - x_1)$$

*When $m > 0$, the line graph is an **increasing function***

*When $m < 0$, the line graph is a **decreasing function***

*When $m = 0$, the line graph is a **constant function***

When the line is vertical 'm' does not exist = no function!

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The Point - Slope Formula

We can plot a straight line using the relationship between any two points on that line:

$$m = (y - y_1) / (x - x_1)$$

We can rewrite this as:

$$(y - y_1) = m (x - x_1)$$

This is the Point-Slope formula that defines an equation of a line with slope m that passes through the point (x_1, y_1)

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The Slope - Intercept Formula

We can also plot a straight line using the more common relationship between any two points on that line:

$$y = m x + b$$

This formula defines an equation of a line with slope m and y -intercept b

Note that these formulas do not define horizontal lines, when $m = 0$:

$$y = b$$

nor vertical lines where $a =$ the x -intercept:

$$x = a$$

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Questions?

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Exam 1 Notes

***Emailed to you not later than 9:00am ET
SUNDAY 3 October 2021***

Exam Duration: One Hour

Must be returned to me same day not later than 1:00pm ET

- Includes Chapters 1 and 2, work covered in class, and/or in your homework assignments.
- Does NOT include Imaginary Numbers

I suggest that you prepare a single, two-sided, page of HANDWRITTEN notes and formulas to use during the exam. You MAY use a graphing calculator during the exam.

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In Future Class Sessions

In class – Session 5:

- **Due:** Homework Set #04 / Quiz#04
- **Lecture:** Creating Equations – Polynomials
- **In class Review:** Exam 1; Homework Sets #03, #04

In class – Session 6:

- **Due:** Homework Set #05 / Quiz#05
- **Lecture:** Polynomial Functions
- **In class Review:** Homework Set #05

In class – Session 7:

- **Due:** Homework Set #06 / Quiz#06
- **Lecture:** Functions, Graphing, Exponents & Logarithms
- **In class Review:** Homework Set #06

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Any Questions?
Send me an email ...

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End

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