Pratt

Math 150 – Fall 2021
Algebra & Trigonometry

Charles Rubenstein, Ph. D.
Professor of Engineering & Information Science

Session 5: Monday 10/04/21

6:30pm - 9:20pm

Online – *Revision 1*

Not Permitted in Class

Be sure to have all cellphones **OFF**

(unless used as calculator…)

Although NOT required

please turn on your cameras

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21 Fall Class Roster : 150-01 (Mon 6:30pm)

MATH150 – 01 Algebra & Trigonometry

Last Name	First Name	Call Me	Time Zone
Garavelo	Naihra	Naihra	ET
Lin	Fanghao	Fanghao	ET
Nguyen	Khanh	Luci	ET
Powers	Tony	Tony	ET
Rakicevic-More	Alek	Alek	ET
Ramirez	Guillermo	Xavier	"- 2"
Richardson	Janie	Janie	ET
Wang	Ke Wei	Ke Wei	ET
Zawadski	Ela	Ela	ET
Zhang	Huiying	Hayley	"+12"

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Instructor Contact Information

Dr. Charles Rubenstein <crubenst@pratt.edu>

Professor of Engineering & Information Science

Brooklyn Campus Faculty Office: ARC G-49

Fall 2021 VIRTUAL Office hours ONLY

Thursdays: **10:00am - 1:00pm** *Via Zoom*

Meeting ID: 569 176 2059

Passcode: Office

To make your appointment Send me an email at least one day in advance:

crubenst@pratt.edu

Subject line: 150 Office Hour

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US Citizens – Student Alert!

To vote in November you must register. Contact **dkahn@nypirg.org**

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Draft Schedule: Math 150 – Fall 2021 – Remote Learning

Monday	Notes
30-Aug	1. Introduction: Numbers, Arithmetic Operations, Fractions
6-Sep	*Pratt Holiday - NO CLASSES – Labor Day*
13-Sep	2. Manipulation of Algebraic Expressions (H/Q1)
20-Sep	3. Solving Linear and Quadratic Equations of One Variable (H/Q2)
27-Sep	4. Solving Equations of Two Variables (H/Q3)
4-Oct	5. Creating Equations – Polynomials (H/Q4); *Exam #1 Sunday 10/3; 9am*
11-Oct	6. Polynomial Functions, continued (H/Q5); *Exam #1 Review*
18-Oct	7. Functions, Graphing, Exponents and Logarithms (H/Q6)
25-Oct	8. Trigonometric Functions, Pythagorean Theorem (H/Q7)
1-Nov	9. Applications of Trigonometry (H/Q8)
8-Nov	10. Analytic Trigonometry, Identities, Graphing (H/Q9) *Exam #2 Sunday 11/7 9am*
15-Nov	11. Areas and Volumes of Geometric Solids (H/Q10) *Exam #2 Review*
22-Nov	12. Systems of Equations and Inequalities
29-Nov	13. Series and Sequences, Review topics
6-Dec	14. Final Examination (3-hour) *Emailed Sunday 12/5 @ 9am due by 2:00pm*

NOTE: Take home exams account for the 15th class session;
Exams emailed Sunday before date noted by 9:00am – due back by 1:00pm

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This Class Session

Exam 1 - One Hour

Was emailed to you not later than 9:00am ET SUNDAY 3 October 2021
And was required to be returned to me not later than 1:00pm ET

In class – Session 5:
- ***Due:* Homework Set #04 / Quiz#04**
- ***Lecture: Creating Equations – Polynomials***
- ***In class Review: Exam 1; Homework Sets #03, #04***

In class – Session 6:
- ***Due:* Homework Set #05 / Quiz#05**
- ***Lecture: Polynomial Functions***
- ***In class Review:* Homework Set #05**

In class – Session 7:
- ***Due:* Homework Set #06 / Quiz#06**
- ***Lecture: Functions, Graphing, Exponents & Logarithms***
- ***In class Review:* Homework Set #06**

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Class Session Archives

www.CharlesRubenstein.com/150

21fa05.pdf *(this slide set)**

21fa05_h.pdf *(slide set as handouts)**

**Available by Thursday evenings…*

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Math 150 – Chapter Topics

1. **The Foundations of Algebra**
2. **Equations and Inequalities**
3. **Functions**
4. **Polynomial Functions**
5. ***Rational Functions and Conic Sections***
6. **Exponential and Logarithmic Functions**
7. **The Trigonometric Functions**
8. **Analytic Trigonometry**
9. **Applications of Trigonometry**
10. ***Systems of Equations and Inequalities***
11. ***Matrices, Linear Systems, and Determinants***
12. ***Topics in Algebra***

(Italicized topics – Ch. 5, 10, 11, and 12 – not always covered...)

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About Your Final Grade

Homework (30%)

For the first **ten (10)** Homework assignments I will select three (3) problems from each assignment to grade. 1% per correct answer, or **3% per homework**

Homework must be emailed to me by 12:00Noon ET on day of class as a doc, rtf, pdf or other file – not as a photo/jpg. The filename MUST be **lastname_hwk##.docx** (*or doc, pdf, etc.*)

Exams (70%)

There will be two (2) one-hour exams worth 20% each

There will be a two-hour FINAL exam worth **30%**

Exams will be emailed to you as noted in the schedule on a Sunday not later than 9:00am ET and must be returned to me not later than 1:00pm ET the same day.

There will be NO make up 'quizzes' or exams

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Questions?

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Exam 1 Notes

Emailed to you not later than 9:00am ET
SUNDAY 3 October 2021
Exam Duration: One Hour
Was to be returned to me same day not later than 1:00pm ET

- **Included Chapters 1 and 2, work covered in class, and/or in your homework assignments.**
- **Did NOT include Imaginary Numbers**

I suggested that you prepare a single, two-sided, page of HANDWRITTEN notes and formulas to use during the exam. You MAY use a graphing calculator during the exam.

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Examination #1
Chapters 1 and 2

REVIEW
NOT ONLINE!
Average grade: 91
Lo: 70 HI: 100

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Questions?

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Homework #03
Selected
Review Problems
Average Quiz Grade: 2.4%

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Homework #3 Review

Ch. 1 Pp 79-82

#17. "A salesperson receives ***3.25x + 0.15y*** *dollars, where* ***x*** *is the number of hours worked and* ***y*** *is the number of miles driven. Find the amount due the salesperson if* ***x = 12 hours*** *and* ***y = 80 miles****."*

\$3.25x + \$0.15y = ?

= \$3.25*(12)* + \$0.15*(80)*

= \$39.00 + \$12.00 = **\$51.00** ← **ans.**

(Clearly this is a problem that does not take into consideration today's \$15+ hourly rates and \$0.58 IRS mileage reimbursement…)

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Homework #3 Review

Ch. 1 Pp 79-82

In Exercises 21–23, perform the indicated operations.

#22. $x(2x-1)(x+2)$

$= (2x^2-x)(x+2)$

$= (2x^3-x^2)+(4x^2-2x)$

$= 2x^3+3x^2-2x$

In Exercises 24–29, factor each expression.

#26. $2a^2+3ab+6a+9b$

$= 2a(a+3) + 3b(a+3)$

$= (2a+3b)(a+3)$

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Homework #3 Review – *Quiz Problem*

Ch. 1 Pp 79-82

#78. *The irrational number called the '**golden ratio**'* $T = \frac{\sqrt{5}+1}{2}$ *has properties that have intrigued artists, philosophers, and mathematicians through the ages.*

Show that **T** satisfies the identity $T = 1 + \frac{1}{T}$

$$\frac{\sqrt{5}+1}{2} = 1 + \frac{1}{\frac{\sqrt{5}+1}{2}}$$

$$1+\frac{1}{\frac{\sqrt{5}+1}{2}} = 1 + \left[\frac{1}{\frac{\sqrt{5}+1}{2}} \cdot \frac{(2)}{(2)}\right]$$

$$= 1 + \frac{2}{\sqrt{5}+1}$$

$$= 1 + \left[\frac{2}{\sqrt{5}+1} \cdot \frac{\sqrt{5}-1}{\sqrt{5}-1}\right]$$

$$= 1 + \frac{2(\sqrt{5}-1)}{5-1}$$

$$= 1 + \frac{2(\sqrt{5}-1)}{4}$$

$$= \frac{4}{4} + \frac{2(\sqrt{5}-2)}{4}$$

$$= \frac{4+2\sqrt{5}-2}{4}$$

$$= \frac{2+2\sqrt{5}}{4}$$

$$= \frac{2(1+\sqrt{5})}{4}$$

$$= \frac{1+\sqrt{5}}{2}$$

$$= \frac{\sqrt{5}+1}{2} = T$$

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Homework #3 Review

Ch. 2.1 (Linear Equations in One Unknown) Pp 93-94

Problems 5, 6, 7, 13, 25, 26, 31, 33

In Exercises 5–24, solve the given linear equation, check your answer.

#5. $3x+5=-1$

$3x=-6$ $x=-2$

#6. $5r+10=0$

$5r=-10$ $r=-2$

#7. $2=3x+4$

$(2-4)=3x$ $-2=3x$ $-2/3=x$

#13. $-5x+8=3x-4$

$-5x-3x=-8-4 \rightarrow -8x=-12$ $x=3/2$

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Homework #3 Review

Solve for x in Exercises 25–28.

#25. $kx+8=5x$

$8=5x-kx \rightarrow 8=x(5-k)$

$8/(5-k)=x$ for $k \neq 5$

#26. $8-2kx=-3x$

$8=2kx-3x \rightarrow 8=x(2k-3)$

$8/(2k-3)=x$ for $k \neq 3/2$

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Homework #3 Review

Solve and check in Exercises 29–44.

#31. $\frac{2}{x}+1=\frac{3}{x}$ $\left(\frac{2}{x}+1\right)(x)=\left(\frac{3}{x}\right)(x)$

$2+x=3$

$x=1$

#33. $\frac{2y-3}{y+3}=\frac{5}{7}$ $\left(\frac{2y-3}{y+3}\right)[7(y+3)]=\left(\frac{5}{7}\right)[7(y+3)]$

$7(2y-3)=5(y+3)$

$14y-21=5y+15$

$9y=36$

$y=4$

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Homework #3 Review

Ch. 2.2 *Pp 103-104 Problems 1, 2, 3, 4, 5, 7, 11, 12*

*In Exercises 1–3, **let n represent the unknown**. Translate from words to an algebraic expression or equation.*

#1. The number of blue chips is **3** more than **twice** the number of red chips.

Number of red chips: *n* Number of blue chips: *3 + 2n*

#2. The number of station wagons on a parking lot is **20** fewer than **3** times the number of sedans.

Number of sedans: n Number of station wagons: 3n – 20

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Homework #3 Review – *Quiz Problem*

Ch. 2.2 *Pp 103-104 Problems 1, 2, 3, 4, 5, 7, 11, 12*

*In Exercises 1–3, **let n represent the unknown**. Translate from words to an algebraic expression or equation.*

#3. **Five less than 6 times** a number is **26**. (*Quiz Problem*)

$(6n-5)=26$

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Homework #3 Review – *Quiz Problem*

In Exercises 4–41, translate from words to an algebraic problem and solve.

#4. Janis is 3 years older than her sister. Thirty years from now the sum of their ages will be 111. Find the current ages of the sisters. *(NOTE: You could solve for either's age first…)*

Who?	Age Now	Age in 30 years
Janis	$x+3$	$(x+3)+30$
Sister	x	$x+30$

$[(x+3)+30]+(x+30)=111$

$2x+63=111$ thus $2x=48$ and

$x=24=$ *sister's age*

$x+3=27=$ *Janis' age*

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Homework #3 Review

In Exercises 4–41, translate from words to an algebraic problem and solve.

#5. John is presently 12 years older than Fred. Four years ago John was twice as old as Fred. How old is each now?

Who?	Age Now	Age 4 years ago
Fred	x	$x - 4$
John	$x+12$	$(x + 12) - 4$

$$(x + 12) – 4 = 2(x – 4)$$

$x + 8 = 2x – 8$ *and thus*

$x = 16$ = *Fred's age*

$x + 12 = 28$ = *John's age*

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Homework #3 Review

In Exercises 4–41, translate from words to an algebraic problem and solve.

#7. Find three consecutive integers whose sum is 21

1st number: x

2nd number: $x + 1$

3rd number: $x + 2$

$$(x) + (x + 1) + (x + 2) = 21$$

$$3x + 3 = 21 \rightarrow 3x = 18$$

and $x = 6$; thus $x + 1 = 7$; $x + 2 = 8$

The three consecutive numbers are 6, 7, and 8.

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Homework #3 Review *- Quiz Problem*

#11. A 12-meter long steel beam is to be cut into two pieces so that one piece will be 4 meters longer than the other.

How long will each piece be?

Length of 1st piece: **x** *and* Length of 2nd piece: **x + 4**

$$(x) + (x + 4) = 12$$

$$2x + 4 = 12 \rightarrow 2x = 8$$

thus; $x = 4$ and $x + 4 = 8$

ANS: The lengths are **4m** and **8m**

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Homework #3 Review

#12. *A rectangular field whose length is 10 meters longer than its width is to be enclosed with exactly 100 meters of fencing material. What are the dimensions of the field?*

Given: Width: **w** Length: **w + 10**

$$2(w) + 2(w + 10) = 100$$

$$4w + 20 = 100 \rightarrow 4w = 80 \rightarrow \text{thus } w = 20$$

therefore, $w + 10 = 30$

The width is ***20m*** *and the length is* ***30m***

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Review Problem

#42. A field consists of a rectangle and a square arranged as shown in the figure below.

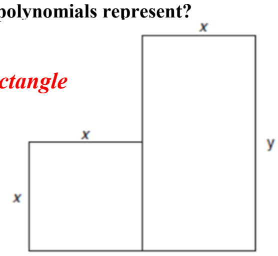
What does each of the following polynomials represent?

a. $x^2 + xy =$

Area of square + area of rectangle = Total Area

b. $2x + 2y =$

Perimeter of the Rectangle



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Questions?

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Homework #04
REVIEW
Section 2.2 and 2.3

Average Quiz Grade: 3%

BUT…

Only 4 students handed in by 4:00pm!

Others will be penalized for lateness…

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Homework #4 Review

2.2 #25. **Pg 105** ***"Professors Roberts and Jones, who live 676 miles apart, are exchanging houses and jobs for the summer. They start out for their new locations at exactly the same time, and they meet after 6.5 hours of driving. If their average speeds differ by 4 mph, what are their average speeds?"***

	rate ·	time =	distance
1st prof	x	6.5	$6.5x$
2nd prof	$x + 4$	6.5	$6.5(x + 4)$

$$6.5x + 6.5(x + 4) = 676$$
$$6.5x + 6.5x + 26 = 676$$
$$13x = 676 - 26 = 650$$
$$x = 50 \text{ and thus } (x + 4) = 54$$

Their speeds are 50 and 54 mph. ← ans.

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Homework #4 Review – *Quiz Problem*

2.2 #28. **Pg 105** ***"How many pounds of raisins worth $3 per pound must be mixed with 10 pounds of peanuts worth $2.40 per pound to produce a mixture worth $2.80 per pound? "***

Pounds of raisins: x

$$3x + 2.40(10) = 2.80(10 + x)$$

Multiply by 10 to remove decimal places:

$$10[3x + 2.40(10)] = 10[2.80(10 + x)]$$
$$30x + 240 = 28(10 + x)$$
$$30x + 240 = 280 + 28x$$
$$2x = 40$$
$$x = 20$$

There should be 20 pounds of raisins in the mixture. ans.

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Homework #4 Review

2.2 #34. **Pg 105** ***"If 1/3 is subtracted from 3 times the reciprocal of a certain number, the result is 25/6. Find the number.?"***

If the number is x:

$$3\left(\frac{1}{x}\right) - \frac{1}{3} = \frac{25}{6}$$
$$6x\left[\frac{3}{x} - \frac{1}{3}\right] = 6x\left(\frac{25}{6}\right)$$
$$18 - 2x = 25x$$
$$18 = 27x$$

x = 2/3 ans.

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Homework #4 Review

2.2 ***In Exercises 42–51 solve for the indicated variable in terms of the remaining variables.***

Pg 105 #42. ***A=Pr solve for r***

$$\frac{1}{P}(A) = \frac{1}{P}(Pr)$$
$$\frac{A}{P} = r$$

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Homework #4 Review

2.2 ***In Exercises 42–51 solve for the indicated variable in terms of the remaining variables.***

Pg 105 #43. ***C=2πr solve for r***

$$\frac{1}{2\pi}(C) = \frac{1}{2\pi}(2\pi r)$$
$$\frac{C}{2\pi} = r$$

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Homework #4 Review

2.2 ***In Exercises 42–51 solve for the indicated variable in terms of the remaining variables.***

Pg 105 #44. ***$V=1/3\ (\pi r^2 h)$ solve for h***

$$\frac{3}{\pi r^2}(V) = \frac{3}{\pi r^2}\left[\frac{1}{3}\pi r^2 h\right]$$

$$\frac{3V}{\pi r^2} = h$$

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Homework #4 Review

2.2 ***In Exercises 42–51 solve for the indicated variable in terms of the remaining variables.***

Pg 105 #45. ***$F= 9/5\ (C) + 32$ solve for C***

$$F - 32 = \frac{9}{5}C$$

$$\frac{5}{9}(F - 32) = C$$

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Homework #4 Review

2.2 ***In Exercises 42–51 solve for the indicated variable in terms of the remaining variables.***

Pg 105 #46. ***$S= ½\ gt^2 + vt$ solve for v***

$$S - \frac{1}{2}gt^2 = vt$$

$$\frac{1}{t}\left(S - \frac{1}{2}gt^2\right) = \frac{1}{t}(vt)$$

$$\frac{S}{t} - \frac{1}{2}gt = v$$

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Homework #4 Review

2.2 ***In Exercises 42–51 solve for the indicated variable in terms of the remaining variables.***

Pg 105 #47. ***$A= ½\ h(b+b')$ solve for b***

$$\frac{2}{h}(A) = \frac{2}{h}\left[\frac{1}{2}h(b+b')\right]$$

$$\frac{2A}{h} = b + b'$$

$$\frac{2A}{h} - b' = b$$

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Homework #4 Review

2.2 ***In Exercises 42–51 solve for the indicated variable in terms of the remaining variables.***

Pg 105 #48. ***$A=P(1+rt)$ solve for r***

$$\frac{A}{P} = 1 + rt$$

$$\frac{A}{P} - 1 = rt$$

$$\frac{A}{Pt} - \frac{1}{t} = r$$

OR

$$\frac{A-P}{Pt} = r$$

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Homework #4 Review

2.2 ***In Exercises 42–51 solve for the indicated variable in terms of the remaining variables.***

Pg 105 #49. ***$1/f = 1/f_1 + 1/f_2$ solve for f_2***

$$\frac{1}{f} - \frac{1}{f_1} = \frac{1}{f_2}$$

$$\frac{f_1 - f}{ff_1} = \frac{1}{f_2}$$

Take reciprocal of both sides.

$$\frac{ff_1}{f_1 - f} = f_2$$

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Homework #4 Review

2.3 **In Exercises 1–14, solve by factoring.**

Pg 122 #3. $x^2 + x - 2 = 0$

$$(x + 2)(x - 1) = 0$$

$$(x + 2) = 0 \text{ or } (x - 1) = 0$$

$$x = -2 \text{ or } x = 1$$

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Homework #4 Review

2.3 **In Exercises 15–24, solve the given equation.**

Pg 122 #16. $4x^2 - 64 = 0$

$$4x^2 = 64$$

$$x^2 = 16$$

$$x = \pm 4$$

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Homework #4 Review – *Quiz Problem*

2.3 **In Exercises 15–24, solve the given equation.**

Pg 122 #18. $6x^2 - 12 = 0$

$$6x^2 = 12$$

$$x^2 = 2$$

$$x = \pm\sqrt{2}$$

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Homework #4 Review

2.3 **In Exercises 25–36, solve by completing the square.**

Pg 122 #25. $x^2 - 2x = 8$

$$\left[\frac{1}{2}(2)\right]^2 = 1$$

$$x^2 - 2x + 1 = 8 + 1$$

$$(x - 1)^2 = 9$$

$$x - 1 = \pm 3$$

$$x = 1 \pm 3$$

$$x = -2 \text{ or } x = 4$$

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Homework #4 Review

2.3 **In Exercises 25–36, solve by completing the square.**

Pg 122 #26. $t^2 - 2t = 15$

$$\left[\frac{1}{2}(2)\right]^2 = 1$$

$$t^2 - 2t + 1 = 15 + 1$$

$$(t - 1)^2 = 16$$

$$t - 1 = \pm 4$$

$$t = 1 \pm 4$$

$$t = -3 \text{ or } t = 5$$

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Homework #4 Review

2.3 **In Exercises 25–36, solve by completing the square.**

Pg 122 #28. $9x^2 + 2x = 2$

$$x^2 + \frac{1}{3}x = \frac{2}{9}$$

$$\left[\frac{1}{2}\left(\frac{1}{3}\right)\right]^2 = \frac{1}{36}$$

$$x^2 + \frac{1}{3}x + \frac{1}{36} = \frac{2}{9} + \frac{1}{36}$$

$$\left(x + \frac{1}{6}\right)^2 = \frac{1}{4}$$

$$x + \frac{1}{6} = \pm\frac{1}{2}$$

$$x = -\frac{1}{6} \pm \frac{1}{2}$$

$$x = \frac{1}{3} \text{ or } x = -\frac{2}{3}$$

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Homework #4 Review

2.3 **In Exercises 25–36, solve by the quadratic formula.**

Pg 122 #37. $2x^2 + 3x = 0$

$a = +2 \; b = +3 \; c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-3 \pm \sqrt{3^2 - 4(2)(0)}}{2(2)}$$
$$= \frac{-3 \pm \sqrt{9}}{4}$$
$$= \frac{-3 \pm 3}{4}$$
$$x = 0 \text{ or } x = -\frac{3}{2}$$

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Homework #4 Review

#40. **Pg 123 "solve by the quadratic formula"**

$2x^2 - 3x - 2 = 0$

Thus: $a = +2 \quad b = -3 \quad c = -2$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(-2)}}{2(2)}$$
$$= \frac{3 \pm \sqrt{25}}{4}$$
$$= \frac{3 \pm 5}{4}$$

Therefore, $x = 2$ **and** $x = -\frac{1}{2}$ ***final answer***

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Homework #4 Review

#50. **Pg 123 "solve by any method"**

$2t^2 + 2t + 3 = 0$

use the quadratic equation: $a = +2 \quad b = +2 \quad c = +3$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-2 \pm \sqrt{2^2 - 4(2)(3)}}{2(2)}$$
$$= \frac{-2 \pm \sqrt{-20}}{4}$$
$$= \frac{-2 \pm 2\sqrt{5}i}{4}$$
$$= \frac{-1 \pm \sqrt{5}i}{2}$$

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Homework #4 Review – *Quiz Problem*

#52. **Pg 123 "solve by any method"**

$x^2 + 2x = 0$

This is a 'special case' factoring:

$x\,(x + 2) = 0$

this becomes $x + 0 = 0; \; x = 0$

and $x + 2 = 0; \; x = -2$

Thus $x = 0$ **or** $x = -2$ ans.

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Questions?

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Math 150 – Chapter Topics

Chapter 4 Polynomial Functions

4.1 Quadratic Functions and Their Graphs
4.2 Graphs of Polynomial Functions of Higher Degree
4.3 Polynomial Division and Synthetic Division
4.4 The Remainder and Factor Theorems
4.5 Factors and Zeros
4.6 Real, Complex and Rational Zeros
4.7 Approximations of the Zeros of a Polynomial Function

Sections 4.2 – 4.7 are not covered in this class
Should you have a question on these sections, please let me know and I will try to do a review of them in future classes.

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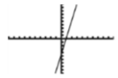
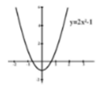
Polynomial Functions

The basic **Polynomial Function** is a Linear function:



$$f(x) = y = ax + b$$

The basic **Quadratic Function** is



$$f(x) = y = ax^2 + bx + c$$

The function becomes the Quadratic Equation when y = 0

In general we can use the following notation for a polynomial equation or function of degree ***n*** with coefficients "a_n" – where we will only concern ourselves with real values of the coefficients and where $a_n \neq 0$:

$$f(x) = y = a_n\, x^n + a_{n-1}\, x^{n-1} + \ldots + a_1\, x^1 + a_0\, x^0$$

Graphing Quadratic Equations

Plotting **Quadratic Equation** of the form

$$f(x) = y = ax^2 + bx + c = 0$$

*where the y-intercept is at "**c**"*

Typically yields roots of the form

$$f(x) = y = (x \pm p)\,(x \pm q)$$

*which correspond to the x-intercepts (where the function passes through the x-axis) that is, where **y = 0***

The three possibilities for quadratic equation roots are:

two different roots,

one double root,

or no real roots

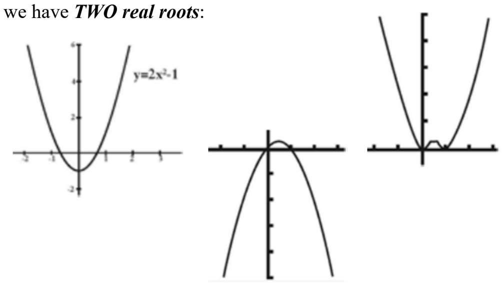
as can be seen on the next three slides…

Graphing Two Root Polynomial Equations

Some **Polynomial Equations** of the form

$$f(x) = y = ax^2 + bx + c = 0$$

result in a graph which crosses the x-axis in TWO places and thus we have ***TWO real roots***:

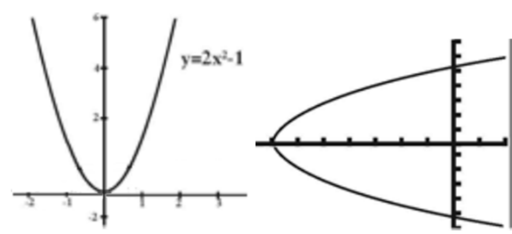


Graphing Double Root Polynomial Equations

Other **Polynomial Equations** of the form

$$f(x) = y = ax^2 + bx + c = 0$$

result in a graph which crosses the x-axis in ONE place and thus we have ***double roots*** - both roots are real and equal to each other:

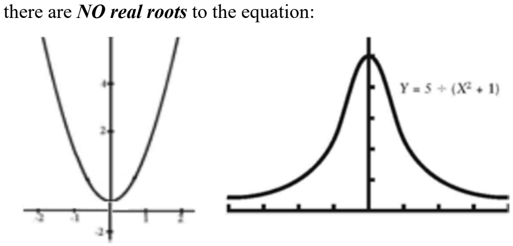


Graphing NO Real Root Polynomial Equations

And there are **Polynomial Equations** of the form

$$f(x) = y = ax^2 + bx + c = 0$$

where the graph DOES NOT cross the x-axis in any place and thus there are ***NO real roots*** to the equation:



Note: CM/FM typically has ONLY two real or double roots…

The Vertex

When we have the **Quadratic Equation**

$$f(x) = y = ax^2 + bx + c = 0$$

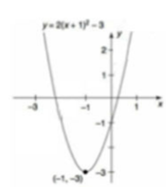
We typically find a parabola which has a maximum of minimum point or y-value. That point is called the VERTEX of the graph.

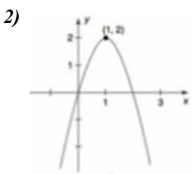
The graph on the left has the equation $f(x) = y = 2(x+1)^2 - 3$

*Its vertex is at **(-1, -3)***

The graph on the right has the equation $f(x) = y = -2(x-1)^2 + 2$

*Its vertex is at **(1, 2)***





The Descriminant

We recall that the roots of the quadratic equation in the form $ax^2 + bx + c = 0$ *for* $a \neq 0$

may be solved using the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Where $b^2 - 4ac$ *is called the* ***descriminant:***

When $b^2 - 4ac$ *is positive → roots are real numbers*

When $b^2 - 4ac$ *is zero → there is a double root*

When the descriminant result is a square, the roots are rational

When $b^2 - 4ac$ *is negative → roots are complex conjugate pairs*

We do not use complex numbers in CM/FM…

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Determinant Examples

We also saw last session that we can express the nature of the roots for a quadratic equation without solving it:

$2x^2 - 3x - 3 = 0$

$b^2 - 4ac$ *is* $(3)^2 - 4(2)(-3) = 9 + 24 = \mathbf{+33}$ *= not a square*

Roots = two real numbers

$4x^2 - 20x + 25 = 0$

$b^2 - 4ac$ *is* $(-20)^2 - 4(4)(25) = 400 - 400 = 0$

Roots = real double roots

$10x^2 = x + 2$

$b^2 - 4ac$ *is* $(-1)^2 - 4(10)(-2) = 1 + 80 = \mathbf{+81}$ *= square*

Roots = two real, rational roots

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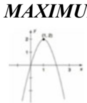
Maximums and Minimums

We can also use the quadratic equation formula to find the value of the graphs maximum or minimum:

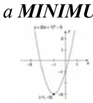
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Where $-b / 2a$ *is the value of x where the maximum or minimum of the graph occurs (at the vertex):*

When $-b / 2a$ *is* ***positive*** *→ the graph has a* ***MAXIMUM*** *value and the parabola opens* ***'down'***



When $-b / 2a$ *is* ***negative*** *→ the graph has a* ***MINIMUM*** *value and the parabola opens* ***'up'***



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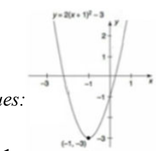
Maximums and Minimums

We looked at these two parabolic graphs earlier when we calculated their vertex values. Now we can also calculate their maximum of minimum values.



The graph has the equation $f(x) = y = 2(x+1)^2 - 3$

Its vertex is at ***(-1, -3)***

and ***y*** *needs to be expanded to find the* ***a*** *and* ***b*** *values:*

$f(x) = y = 2(x+1)^2 - 3 = 2(x+1)(x+1) - 3 =$

$2(x^2 + 2x + 1) - 3 = (2x^2 + 4x + 2) - 3 = 2x^2 + 4x - 1$

Thus the ***max/min*** $= -b/2a = -4/(2(2)) = -4/4 = -1$

The ***negative value*** *notes the parabola has a* ***MINIMUM*** *and that it opens* ***'up'***

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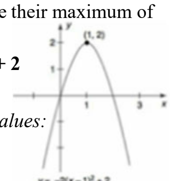
Maximums and Minimums

We looked at these two parabolic graphs earlier when we calculated their vertex values. Now we can also calculate their maximum of minimum values.



The graph has the equation $f(x) = y = -2(x-1)^2 + 2$

Its vertex is at ***(1, 2)***

and ***y*** *needs to be expanded to find the* ***a*** *and* ***b*** *values:*

$f(x) = y = -2(x-1)^2 + 2 = -2(x-1)(x-1) + 2 =$

$-2(x^2 - 2x + 1) + 2 = (-2x^2 + 4x - 2) + 2 = -2x^2 + 4x$

Thus the ***max/min*** $= -b/2a = -4/(2(-2)) = -4/-4 = +1$

The ***positive value*** *notes the parabola has a* ***MAXIMUM*** *and that it opens* ***'down'***

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Math 150 – Chapter Topics

Chapter 5. Rational Functions and Conic Sections

5.1 Rational Functions and Their Graphs
5.2 The Circle
5.3 The Parabola
5.4 The Ellipse and Hyperbola
5.5 Translation of Axes

Sections 5.1 – 5.5 are not covered in this class

Should you have a question on these sections, please let me know and I will try to do a review of them in future classes.

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Questions?

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In Future Class Sessions

In class **– Session 6:**

- ***Due and Reviewed:*** ***Homework Set #05 / Quiz#05***
- ***Lecture: Polynomial Functions***
- ***In Class Review:*** ***Homework Set #05 in class***

In class **– Session 7:**

- ***Due and Reviewed :*** ***Homework Set #06 / Quiz#06***
- ***Lecture: Functions, Graphing, Exponents & Logarithms***
- ***In Class Review:*** ***Homework Set #06 in class***

In class **– Session 8:**

- ***Due and Reviewed :*** ***Homework Set #07 / Quiz#07***
- ***Lecture: Trigonometric Functions, Pythagorean Theorem***
- ***In Class Review:*** ***Homework Set #07 in class***

In class **– Session 9:**

- ***Due and Reviewed :*** ***Homework Set #08 / Quiz#08***
- ***Lecture: Applications of Trigonometry***
- ***In Class Review:*** ***Homework Set #08 in class***

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Any Questions?
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End

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